7.3 Funciones exponenciales.
Cada ejercicio tiene su valor en [].
Tarea 9 (obligatoria) Calcular la derivada de las siguientes funciones:

1. [2.0]
$$\tan(x)^{\sin\left(x^2\right)}.$$

RESPUESTA. $y = \tan(x)^{\sin\left(x^2\right)}$.

$\ln(y) = \ln\left(\tan(x)^{\sin\left(x^2\right)}\right) = \sin\left(x^2\right)\ln(\tan(x))$

$\frac{d}{dx}\left(\tan(x)^{\sin\left(x^2\right)}\right)$.

$\frac{d}{dx}\ln(y) = \frac{y'}{y}$.

$\frac{d}{dx}\left(\sin\left(x^2\right)\ln(\tan(x))\right) = \left(\sin\left(x^2\right)\right)'\ln(\tan(x)) + \sin\left(x^2\right)(\ln(\tan(x)))' =$
$\cos\left(x^2\right)2x\ln(\tan(x)) + \sin\left(x^2\right)\frac{1}{\tan x}\sec^2 x$.

$\frac{y'}{y} = \cos\left(x^2\right)2x\ln(\tan(x)) + \sin\left(x^2\right)\frac{1}{\tan x}\sec^2 x$, sust. y.

$\frac{d}{dx}\left(\tan(x)^{\sin\left(x^2\right)}\right) = \tan(x)^{\sin\left(x^2\right)}\left(2x\cos\left(x^2\right)\ln(\tan(x)) + \frac{\sin\left(x^2\right)\sec^2 x}{\tan x}\right)$.

Otro método
$\frac{d}{dx}\exp\left(\ln\left(\tan(x)^{\sin\left(x^2\right)}\right)\right) = \frac{d}{dx}\left(\exp\left(\sin x^2\ln(\tan(x))\right)\right) =$
$\tan(x)^{\sin\left(x^2\right)}\left(\cos\left(x^2\right)2x\ln(\tan(x)) + \sin\left(x^2\right)\frac{1}{\tan x}\sec^2 x\right)$.

2. [2.0]
$$\left(5^x + x^2\right)^{\cos(\pi x)}.$$

RESPUESTA.
$\frac{d}{dx}\left(\left(5^x + x^2\right)^{\cos(\pi x)}\right) =$
$\left(x^2 + 5^x\right)^{\cos\pi x - 1}\left(2x\cos\pi x + 5^x\ln 5\cos\pi x - \pi x^2\ln\left(x^2 + 5^x\right)\sin\pi x - 5^x\pi\ln\left(x^2 + 5^x\right)\sin\pi x\right)$.

3. [2.0]
$$\frac{x\tan\left(x + x^2\right)}{\left(\cos^2(x) + \sin^2(x)\right)^{\left(\frac{x-5}{x^2-2}\right)}}.$$

RESPUESTA.
$\frac{d}{dx}\left(\frac{x\tan\left(x+x^2\right)}{\left(\cos^2(x)+\sin^2(x)\right)^{\left(\frac{x-5}{x^2-2}\right)}}\right) = \frac{d}{dx}\left(x\tan\left(x + x^2\right)\right) = 2x^2\tan^2 x(x+1) + 2x^2 + x\tan^2 x(x+1) + x +$
$\tan x(x+1)$.

Note $\frac{d}{dx}\left(\tan\left(x + x^2\right)\right) = \left(\tan^2 x(x+1) + 1\right)(2x+1)$.

$\frac{d}{dx}(\tan(x)) = \tan^2 x + 1 = \sec^2 x$.

Porqué $\tan^2 x + 1 = \frac{\sin^2 x}{\cos^2 x} + 1 = \frac{\sin^2 x + \cos^2 x}{\cos^2 x}1 = \sec^2 x$.

4. [1.0]
$$x^{2\ln(x)}.$$

RESPUESTA.
$\frac{d}{dx}\left(x^{2\ln(x)}\right) = 4x^{2\ln x - 1}\ln x$.

5. [3.0]

$$\exp\left(\exp\left(x^{4\ln(x)}\right)\right).$$

RESPUESTA.

$\frac{d}{dx}\left(\exp\left(\exp\left(x^{4\ln(x)}\right)\right)\right) = 8x^{4\ln x - 1}e^{x^{4\ln x}}e^{e^{x^{4\ln x}}}\ln x.$