$$[\ln(y)]' = [\ln(x^z)]'$$
$$[z \cdot \ln(x)]'$$
$$\frac{1}{y} y' = z\,[\ln(x)]'$$
$$= z \; 1/x$$
$$y' = \frac{z}{x} y, \text{ sust } y$$
$$y' = \frac{z}{x} x^z = z x^z x^{-1}$$
$$= z x^{z-1} = z x^{z-1}$$

$$\boxed{\frac{dx^n}{dx} = n x^{n-1}}$$

Límites Indeterminadas

$$\lim_{\theta \to \infty} \frac{\operatorname{sen}\theta}{\theta} = \frac{\infty}{\infty}$$
$$\lim_{\theta \to 0} \frac{\operatorname{sen}\theta}{\theta} = \frac{0}{0}$$
$$\lim_{\theta \to \infty} \frac{e^x}{x^x} = \frac{\infty}{\infty}$$

← No se pueden calcular por sustitución Directa

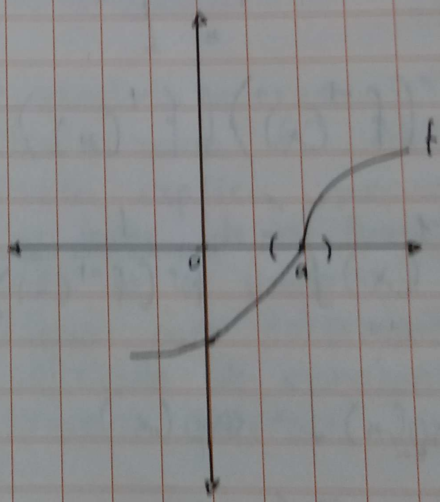
Regla de L'Hôpital

Una versión del Teorema de L'Hopital dice

- Sean f, g, funciones continuas y con derivada en el punto interior a con f(a) = g(a) = 0

Entonces:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} \ldots$$
$$\lim_{x \to a} \frac{f''(x)}{g''(x)}$$



DEMOSTRAR

- Demostración: Por las hipótesis podemos aproximar f y g por un desarrollo de Taylor de de primer orden, en el punto a.

$$f(x) = \sum_{k=0}^{n} \frac{f^{k}(a)}{k!}(x-a)^{k} + \mathcal{O}(\ \ )$$

$$= f(a) + f'(a)(x-a) + \mathcal{O}(x-a)^2$$

$$f(x) = f(a) + f'(a)(x-a)$$
$$g(x) = g(a) + f'(a)(x-a)$$

Sust. en

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{\cancel{f(a)} + f'(a)(x-a) + \mathcal{O}(x-a)^2}{\cancel{g(a)} + g'(a)(x-a) + \mathcal{O}(x-a)^2}$$

$$= \lim_{x\to a}\frac{f'(a)(x-a) + \mathcal{O}(x-a)^2}{g'(a)(x-a) + \mathcal{O}(x-a)^2}$$

$$= \frac{\lim_{x\to a} f'(a)(x-a) + \mathcal{O}(x-a)^2}{\lim_{x\to a} g'(a)(x-a) + \mathcal{O}(x-a)^2}$$

$$= \frac{\lim_{x\to a} f(a)(x-a) + \lim_{x\to a}\mathcal{O}(x-a)^2}{\lim_{x\to a} g'(a)(x-a) + \lim_{x\to a}\mathcal{O}(x-a)^2}$$

$$= \frac{\lim_{x\to a} f'(a)(x-a)}{\lim_{x\to a} g'(a)(x-a)} = \frac{f'(a)}{g'(a)}$$

Ejemplo 1

① $\lim_{\theta\to 0}\frac{\operatorname{sen}\theta}{\theta} = \frac{0}{0}$

Respuesta por L'Hôpital

$$\lim_{\theta\to 0}\frac{\operatorname{sen}\theta}{\theta} = \lim_{\theta\to 0}\frac{\cos\theta}{1} = 1$$

donde $f = \operatorname{sen}\theta$ $\quad g = \theta$
$f' = \cos\theta$ $\quad g' = 1$

Otra versión

$$\lim_{\theta\to 0}\frac{\operatorname{sen}\theta}{\theta}$$

Desarrollo de Taylor (Maclaurin) de primer orden para $\operatorname{sen}(\theta)$ con $\theta = 0$

$$\operatorname{Sen}(\theta) = \sum_{k=0}^{n}\frac{f^{(k)}(0)}{k!}\theta$$

$$= f(0) + f'(0)$$

$$= \operatorname{sen}(0) + \cos(0)\,\theta$$
$$= 0 + 1(0) = \theta$$

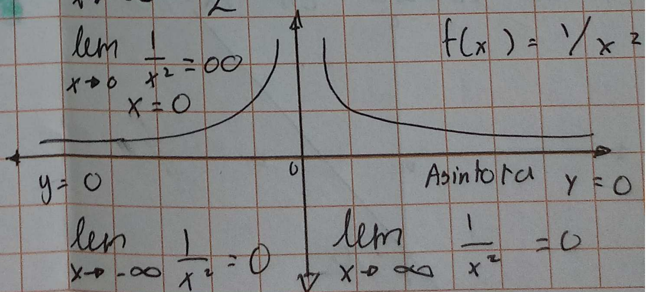
$$\lim_{\theta\to 0}\frac{\operatorname{sen}\theta}{\theta} = \lim_{\theta\to 0}\frac{\theta}{\theta} = 1$$

Ejemplo 2

$$\lim_{x\to\infty}\frac{e^x}{x^2} = \lim_{x\to\infty}\frac{e^x}{2x} =$$

$$\lim_{x\to\infty}\frac{e^x}{2} = \infty$$

$$\lim_{x\to 0}\frac{1}{x^2} = \infty$$
$x = 0$

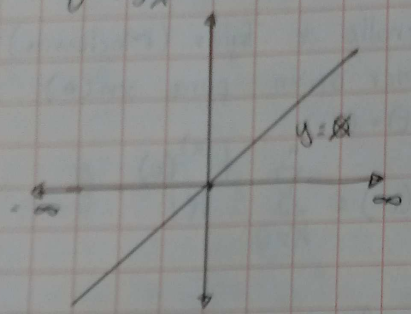


$$\lim_{x\to -\infty}\frac{1}{x^2} = 0 \qquad \lim_{x\to\infty}\frac{1}{x^2} = 0$$

Asintotas Oblicuas

$$f(x) = \frac{x^3 - 2x}{x^2 + 1}$$

$$\begin{array}{r|l} & x \\ x^2+1 & x^3 - 2x \\ & -x^3 - x \\ \hline & 0 \; -3x \end{array}$$



$$\begin{array}{r|l} & x \\ x^2+1 & x^3 - 2x \\ & -x^3 - x \\ \hline & 0 \; -3x \end{array}$$

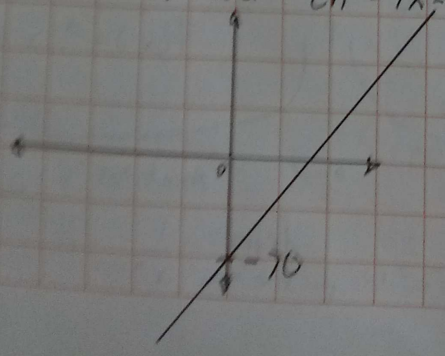
$$\lim_{x \to \infty} \frac{3x}{x^2+1} = 0$$

$$\lim_{x \to} \frac{3}{x + 1/x} = 0$$

$$g(x) = \frac{-7x^2 - 2}{x - 10} = -7x + \frac{70x + 2}{x - 10}$$

$$\lim_{x \to \infty} \frac{70x+2}{x-10} = \frac{70 + 2/x}{1 - 10/x} = 70$$

Asintota Oblicua en $-7x - 70$



Lluvia de Ejemplos!

$$\lim_{x \to 0} \frac{3x - \operatorname{sen} x}{x}$$

$$\lim_{x \to 0} \frac{3 - \cos x}{1} = 2$$

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$$\lim_{x \to 0} 1/2 \, (1+x)^{-1/2}$$

$$\lim_{x \to 0} \frac{1}{2} \, \frac{1}{(1+x)^{1/2}} = 1/2$$

$$\lim_{x \to 0} \frac{x - \sin x}{x^3} =$$

$$\lim_{x \to 0} \frac{1 - \cos x}{3x^2} =$$

$$\lim_{x \to 0} \frac{\operatorname{sen} x}{6x} =$$

$$\lim_{x \to 0} \frac{\cos x}{6} = 1/6$$

$$\lim_{x \to \pi/2} \frac{\operatorname{sen} x}{1 + \tan x}$$

$$\lim_{x \to \pi/2} \frac{\sec x \tan x}{\sec^2 x}$$

$$\lim_{x \to \pi/2} \frac{1/\cos \cdot \operatorname{sen} x/\cos x}{1/\cos^2 x}$$

c $\lim_{x \to \pi/2} \sin x = \sin(\pi/2) = 1$

d $\lim_{x \to \infty} \frac{\ln(x)}{2\sqrt{x}}$

$\lim_{x \to \infty} \frac{1/x}{1\, x^{-1/2}} = \lim_{x \to \infty} x^{1/2} x^{-1}$

$\lim_{x \to \infty} x^{-1/2} = \lim_{x \to \infty} \frac{1}{\sqrt{x}}$

$= 0$

e $\lim_{x \to \infty} \frac{e^{2x}}{x^2} =$

$\lim_{x \to \infty} \frac{\cancel{2} e^{2x}}{\cancel{2} x}$

$\lim_{x \to \infty} \frac{2 e^{2x}}{1} = \infty$