# Collocation Method

Now, taking $p(x) = A\sin\left(\frac{\pi}{H}x\right) \in V$ and substitute it in $(S2)$,

$$E\frac{d^2}{dx^2}\left(A\sin\left(\frac{\pi}{H}x\right)\right) - f_0,$$

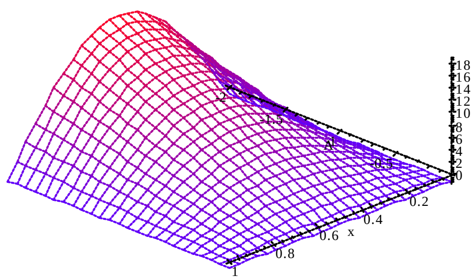
the residual equation is obtained:

$$R\left(A,x\right) = -\frac{EA\pi^2}{H^2}\left(\sin\frac{\pi}{H}x\right) - f_0.$$

The collocation method requires that the residual equation of the aproximate solution be zero at as many points as there are undetermined parameters. This example has only one parameter $A$, the point $x = \frac{H}{2}$ is selected, then $R\left(A,\frac{H}{2}\right) = -\frac{EA\pi^2}{H^2}\left(\sin\frac{\pi}{H}\frac{H}{2}\right) - f_0 = 0$. Therefore the solution is : $A = -\frac{H^2 f_0}{E\pi^2}$.

Desarrolo en clase

$E\frac{d^2}{dx^2}\left(A\sin\left(\frac{\pi}{H}x\right)\right) - f_0 = -EA\left(\sin\frac{\pi}{H}x\right)\frac{\pi^2}{H^2} - f_0$



$R\left(A,x\right) = -EA\left(\sin\frac{\pi}{H}x\right)\frac{\pi^2}{H^2} - f_0$ Grafica del residuo

$R\left(A,\frac{H}{2}\right) = -EA\frac{\pi^2}{H^2} - f_0 = 0$

$-EA\frac{\pi^2}{H^2} - f_0 = 0$, Solution is : $\left\{A = -f_0\frac{H^2}{E\pi^2}\right\}$

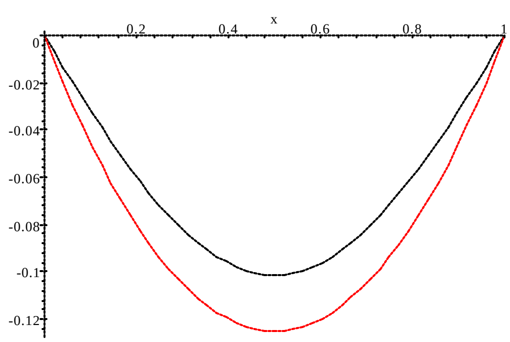
Solución exacta:

$p(x) = f_0\frac{x}{2E}\left(x - H\right)$

$\frac{x}{2}\left(x-1\right)$

$p_c(x) = -f_0\frac{H^2}{E\pi^2}\sin\left(\frac{\pi}{H}x\right)$

Comparación entre la solución exacta y la obtenida por colocación.




$-\frac{1}{\pi^2}\sin(\pi x)$

$E\frac{d^2}{dx^2}\left(-f_0\frac{H^2}{E\pi^2}\sin\left(\frac{\pi}{H}x\right)\right) = f_0\sin\frac{\pi}{H}x$

$f_0 \sin\frac{\pi}{H}\frac{H}{2} - f_0 = 0.$

Otro punto

$x = \frac{H}{4}$

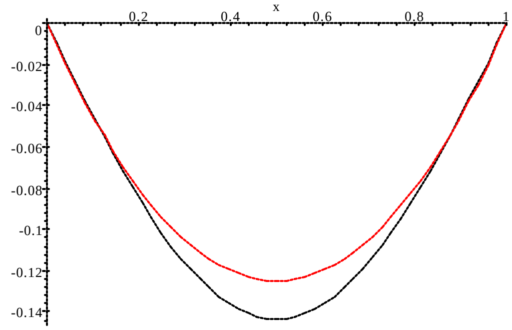
$R\left(A, \frac{H}{4}\right) = -\frac{1}{2}EA\sqrt{2}\frac{\pi^2}{H^2} - f_0$

$-\frac{1}{2}EA\sqrt{2}\frac{\pi^2}{H^2} - f_0 = 0$, Solution is : $\left\{A = -f_0\frac{H^2}{E}\frac{\sqrt{2}}{\pi^2}\right\}$

$f_0 = 1$

$E = 1$

$H = 1$

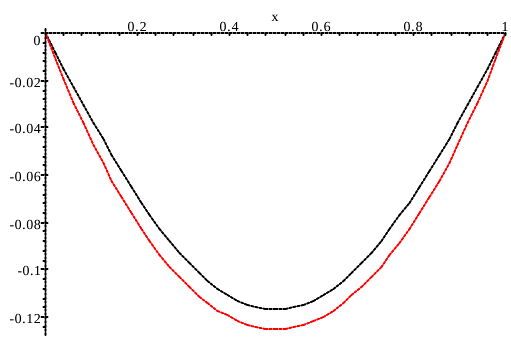



$p_2(x) = -f_0\frac{H^2}{E}\frac{\sqrt{2}}{\pi^2}\sin\left(\frac{\pi}{H}x\right)$ Grafica de la solución exacta y la del punto $\frac{H}{4}$.

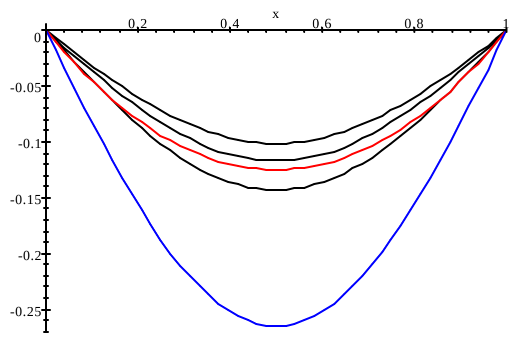
Otro punto.

$R\left(A, \frac{H}{3}\right) = -\frac{1}{2}A\sqrt{3}\pi^2 - 1$

$-\frac{1}{2}A\sqrt{3}\pi^2 - 1 = 0$, Solution is : $\left\{A = -\frac{2}{3}\frac{\sqrt{3}}{\pi^2}\right\}$




$p_3(x) = -\frac{2}{3}\frac{\sqrt{3}}{\pi^2}\sin\left(\frac{\pi}{H}x\right)$ Grafica de la solución exacta y la del punto $\frac{H}{3}$.




$p(x), p_c(x), p_2(x), p_3(x)$

$$R\left(A, \frac{H}{8}\right) = -\frac{1}{2}A\sqrt{\left(2-\sqrt{2}\right)}\pi^2 - 1$$

$$-\frac{1}{2}A\sqrt{\left(2-\sqrt{2}\right)}\pi^2 - 1 = 0, \text{ Solution is : } \left\{A = -\frac{2}{\sqrt{\left(2-\sqrt{2}\right)}\pi^2}\right\}$$

$$p_4(x) = -\frac{2}{\sqrt{\left(2-\sqrt{2}\right)}\pi^2}\sin\left(\frac{\pi}{H}x\right)$$

# Subdomain Method

In this method the integral of the residual equation is aproximate to zero as many subintervals as thera are undetermined parameters. In this example there are only one parameter, therefore the complete interval is used:

$$\int_0^H R(x)dx = \int_0^H \left(-\frac{EA\pi^2}{H^2}\left(\sin\frac{\pi}{H}x\right) - f_0\right)dx = 0.$$

Metodo de subdominios.

para el problema de juego

$E\frac{d^2}{dx^2}(p(x)) - f_0 = 0, 0 < x < H,$

$p(0) = p(H) = 0.$

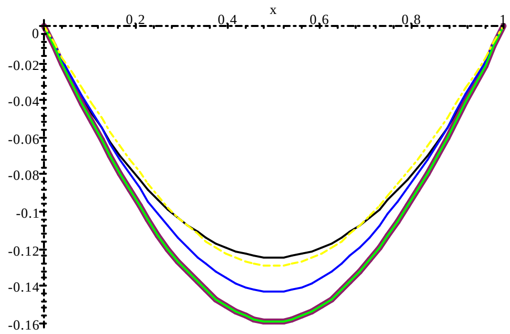
se tiene el residuo

$R(A,x) = -EA\left(\sin\frac{\pi}{H}x\right)\frac{\pi^2}{H^2} - f_0$

$\int_0^H \left(-EA\left(\sin\frac{\pi}{H}x\right)\frac{\pi^2}{H^2} - f_0\right)dx = -\frac{2\pi EA + f_0H^2}{H} = -2\pi A - 1$

$-\frac{2\pi EA + f_0H^2}{H} = 0$, Solution is : $\left\{A = -\frac{1}{2}f_0\frac{H^2}{\pi E}\right\}$

$p_s(x) = -\frac{1}{2}f_0\frac{H^2}{\pi E}\sin\left(\frac{\pi}{H}x\right)$




$p(x)$

Note que se enciman las soluciones $p_s$ y $p_5$.

$\int_0^H (p(x) - p_s(x))dx = 1.7988 \times 10^{-2}$
$\int_0^H (p(x) - p_2(x))dx = 7.8878 \times 10^{-3}$

Otro subdominio

$$\int_0^{H/2} \left(-EA\left(\sin\frac{\pi}{H}x\right)\frac{\pi^2}{H^2} - f_0\right)dx = -\frac{1}{2} - \pi A$$

$-\frac{1}{2} - \pi A = 0$, Solution is : $\left\{A = -\frac{1}{2\pi}\right\}$

$$p_5(x) = -\frac{1}{2\pi}\sin\left(\frac{\pi}{H}x\right)$$

Otro subdominio

$$\int_{H/8}^{7H/8} \left(-EA\left(\sin\frac{\pi}{H}x\right)\frac{\pi^2}{H^2} - f_0\right)dx = -\sqrt{\left(2+\sqrt{2}\right)}\pi A - \frac{3}{4}$$

$-\sqrt{\left(2+\sqrt{2}\right)}\pi A - \frac{3}{4} = 0$, Solution is : $\left\{A = -\frac{3}{4\sqrt{\left(2+\sqrt{2}\right)}\pi}\right\}$

$$p_6(x) = -\frac{3}{4\sqrt{\left(2+\sqrt{2}\right)}\pi}\sin\left(\frac{\pi}{H}x\right)$$

$\int_0^H (p(x) - p_6(x))dx = -1.0814 \times 10^{-3}$ y esta es la mejor solución hasta el momento.