Tercer examen parcial de Cálculo Diferencial e Integral II

Trimestre 11-O

RESPUESTAS

1. (**25** %) Resuelva la integral

$$\int \sqrt{8x - 4x^2}\, dx$$

RESPUESTA

$\int \sqrt{8x - 4x^2}\, dx = \int \sqrt{-4 + 4 + 8x - 4x^2}\, dx =$

$-4 + 4 + 8x - 4x^2 = 4 - (2x-2)^2$

$\int \sqrt{(4 - (2x-2)^2}\, dx =$

$u = 2x - 2; du = 2dx; dx = \frac{du}{2}$

$\frac{1}{2}\int \sqrt{4 - u^2}\, du = \frac{1}{2}\left(\frac{u}{2}\sqrt{4 - u^2} + \frac{4}{2}\sin^{-1}(\frac{u}{2})\right) + C =$

$\frac{1}{2}\left(\frac{2x-2}{2}\sqrt{4 - (2x-2)^2} + 2\sin^{-1}(\frac{2x-2}{2})\right) + C =$

$(x-1)\sqrt{1 - (x-1)^2} + \sin^{-1}(x-1) + C$

Sugerencia: Complete cuadrados en el argumento de la raiz cuadrada que conforma el integrando

2. (**25** %) Calcule la integral

$$\int \frac{13x^2 - 2x + 19}{(x-1)(x^2 + 2x + 3)} dx$$

RESPUESTA.

$\int \frac{13x^2-2x+19}{(x-1)(x^2+4)} dx$

Por fracciones parciales

$\frac{13x^2-2x+19}{(x-1)(2x+x^2+3)} = \frac{5}{(x-1)} + \frac{8x-4}{(x^2+2x+3)}$

Porque

$13x^2 - 2x + 19 = A(x^2 + 2x + 3) + (Bx + C)(x-1) = Ax^2 + 2Ax + 3A + Bx^2 - Bx + Cx - C$

$13x^2 - 2x + 19 = x^2(A + B) + x(2A - B + C) + 3A - C;$

$13 = A + B; (1)$

$-2 = 2A - B + C; (2)$

$19 = 3A - C; (3)$

$(2) + (3) \Rightarrow 17 = 5A - B; 13 = A + B; (1); \Rightarrow 30 = 6A; \Rightarrow \mathbf{A} = \mathbf{5}$

$19 = 3(5) - C; \Rightarrow C = 15 - 19 = -4; \mathbf{C} = \mathbf{-4}$

$13 = 5 + B; \Rightarrow \mathbf{B} = \mathbf{8}$

$\int \frac{13x^2-2x+19}{(x-1)(2x+x^2+3)} dx = \int\left(\frac{5}{(x-1)} + \frac{8x-4}{(x^2+2x+3)}\right)dx =$

$\int \frac{5}{(x-1)} dx + 4\int \frac{2x-1}{(x^2+2x+3)} dx =$

$5\ln|x-1| + 4\int \frac{2x+2-3}{(x^2+2x+3)} dx$

$4\int \frac{2x+2-3}{(x^2+2x+3)} dx = 4\int \frac{2x+2}{(x^2+2x+3)} dx - 12\int \frac{1}{(x^2+2x+3)} dx$

$u = x^2 + 2x + 3; du = (2x+2)dx; x^2 + 2x + 3 = (x+1)^2 + 2; v = x + 1; dv = dx$

$4\int \frac{1}{u}dx - 12\int \frac{1}{v^2+2}dv = 4\ln(u) - 12\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{v}{\sqrt{2}}\right) + C =$

$4\ln|x^2+2x+3| - 12\left(\frac{1}{\sqrt{2}}\tan^{-1}\frac{x+1}{\sqrt{2}}\right) + C$

la solucion completa es

$5\ln|x-1| + 4\ln|x^2+2x+3| - 6\sqrt{2}\tan^{-1}\frac{x+1}{\sqrt{2}} + C$

3. (**25** %) Encuentre la siguiente integral impropia

$$\int_2^3 \frac{x^2}{\sqrt{x^2-4}}dx$$

RESPUESTA.

$\int \frac{x^2}{\sqrt{x^2-4}}dx = 2\ln\left(2x+2\sqrt{x^2-4}\right) + \frac{1}{2}x\sqrt{x^2-4}$

$2\ln\left(2(3)+2\sqrt{(3)^2-4}\right) + \frac{1}{2}(3)\sqrt{(3)^2-4} - \lim_{x\to 2}\left(2\ln\left(2x+2\sqrt{x^2-4}\right) + \frac{1}{2}x\sqrt{x^2-4}\right)$

$2\ln\left(6+2\sqrt{9-4}\right) + \frac{3}{2}\sqrt{9-4} - \left(2\ln\left(2(2)+2\sqrt{(2)^2-4}\right) + \frac{1}{2}(2)\sqrt{(2)^2-4}\right) =$

$2\ln\left(6+2\sqrt{5}\right) + \frac{3}{2}\sqrt{5} - 2\ln(4) \approx 5.2789$

4. (**25** %) Obtener el polinomio de Mclaurin de grado 6 para la función $\cos(x)$ y utilizarlo para calcular

$$\int_{10^{-5}}^{0.1} \frac{\cos(x^2)}{x^2}dx$$

RESPUESTA

$\cos(x) = \cos(0) - \sin(0)x - \frac{1}{2!}\cos(0)x^2 + \frac{1}{3!}\sin(0)x^3 + \frac{1}{4!}\cos(0)x^4 - \frac{1}{5!}\sin(0)x^5 - \frac{1}{6!}\cos$

$\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6;$

$\cos(x^2) = 1 - \frac{1}{2!}x^4 + \frac{1}{4!}x^8 - \frac{1}{6!}x^{12}$

$\int_{10^{-5}}^{0.1} \frac{\cos(x^2)}{x^2}dx \approx \int_{10^{-5}}^{1} \frac{1}{x^2}\left(1 - \frac{1}{2!}x^4 + \frac{1}{4!}x^8 - \frac{1}{6!}x^{12}\right)dx =$

$\int_{10^{-5}}^{0.1}\left(\frac{1}{x^2} - \frac{1}{2!}x^2 + \frac{1}{4!}x^6 - \frac{1}{6!}x^{10}\right)dx =$

$\int_{10^{-5}}^{0.1}\left(\frac{1}{x^2} - \frac{1}{2!}x^2 + \frac{1}{4!}x^6 - \frac{1}{6!}x^{10}\right)dx = \left[-\frac{1}{x} - \frac{1}{(3)2!}x^3 + \frac{1}{(7)4!}x^7 - \frac{1}{(11)6!}x^{11}\right]_{10^{-5}}^{0.1} =$

$\left[-10 - \frac{10^{-3}}{(3)2!} + \frac{10^{-7}}{(7)4!} - \frac{10^{-11}}{(11)6!}\right] - \left[-\frac{1}{10^{-5}} - \frac{1}{(3)2!}10^{-15} + \frac{1}{(7)4!}10^{-35} - \frac{1}{(11)6!}10^{-55}\right] =$

$-10 - \frac{10^{-2}}{(3)2!} + 100000 =$

$99990 - \frac{10^{-2}}{6} = 99989.99833$