DERIVADAS E INTEGRALES

Reglas básicas de derivación

1. $\frac{d}{dx}[cu] = cu'$
2. $\frac{d}{dx}[u \pm v] = u' \pm v'$
3. $\frac{d}{dx}[uv] = uv' + vu'$
4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$
5. $\frac{d}{dx}[c] = 0$
6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$
7. $\frac{d}{dx}[x] = 1$
8. $\frac{d}{dx}[|u|] = \frac{u}{|u|}(u'),\ u \neq 0$
9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
10. $\frac{d}{dx}[e^u] = e^u u'$
11. $\frac{d}{dx}[\operatorname{sen} u] = (\cos u)u'$
12. $\frac{d}{dx}[\cos u] = -(\operatorname{sen} u)u'$
13. $\frac{d}{dx}[\operatorname{tg} u] = (\sec^2 u)u'$
14. $\frac{d}{dx}[\operatorname{ctg} u] = -(\operatorname{cosec}^2 u)u'$
15. $\frac{d}{dx}[\sec u] = (\sec u \operatorname{tg} u)u'$
16. $\frac{d}{dx}[\operatorname{cosec} u] = -(\operatorname{cosec} u \operatorname{ctg} u)u'$
17. $\frac{d}{dx}[\operatorname{arcsen} u] = \frac{u'}{\sqrt{1 - u^2}}$
18. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1 - u^2}}$
19. $\frac{d}{dx}[\operatorname{arctg} u] = \frac{u'}{1 + u^2}$
20. $\frac{d}{dx}[\operatorname{arcctg} u] = \frac{-u'}{1 + u^2}$
21. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{|u|\sqrt{u^2 - 1}}$
22. $\frac{d}{dx}[\operatorname{arccosec} u] = \frac{-u'}{|u|\sqrt{u^2 - 1}}$

Fórmulas básicas de integración

1. $\int kf(u)\,du = k\int f(u)\,du$
2. $\int [f(u) \pm g(u)]\,du = \int f(u)\,du \pm \int g(u)\,du$
3. $\int du = u + C$
4. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$
5. $\int \frac{du}{u} = \ln|u| + C$
6. $\int e^u\,du = e^u + C$
7. $\int \operatorname{sen} u\,du = -\cos u + C$
8. $\int \cos u\,du = \operatorname{sen} u + C$
9. $\int \operatorname{tg} u\,du = -\ln|\cos u| + C$
10. $\int \operatorname{ctg} u\,du = \ln|\operatorname{sen} u| + C$
11. $\int \sec u\,du = \ln|\sec u + |\operatorname{tg} u| + C$
12. $\int \operatorname{cosec} u\,du = -\ln|\operatorname{cosec} u + \operatorname{ctg} u| + C$
13. $\int \sec^2 u\,du = \operatorname{tg} u + C$
14. $\int \operatorname{cosec}^2 u\,du = -\operatorname{ctg} u + C$
15. $\int \sec u \operatorname{tg} u\,du = \sec u + C$
16. $\int \operatorname{cosec} u \operatorname{ctg} u\,du = -\operatorname{cosec} u + C$
17. $\int \frac{du}{\sqrt{a^2 - u^2}} = \operatorname{arcsen}\frac{u}{a} + C$
18. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\operatorname{arctg}\frac{u}{a} + C$
19. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\operatorname{arcsec}\frac{|u|}{a} + C$

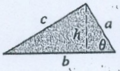
FORMULAS DE GEOMETRIA

Triángulo

$h = a \operatorname{sen} \theta$

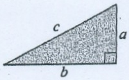
Area $= \frac{1}{2} bh$

(Teorema del coseno)
$c^2 = a^2 + b^2 - 2ab \cos \theta$



Triángulo rectángulo

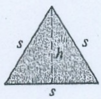
(Teorema de Pitágoras)
$c^2 = a^2 + b^2$



Triángulo equilátero

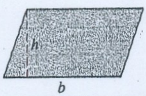
$h = \frac{\sqrt{3}s}{2}$

Area $= \frac{\sqrt{3}s^2}{4}$



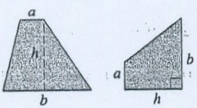
Paralelogramo

Area $= bh$



Trapecio

Area $= \frac{h}{2}(a + b)$



Círculo

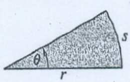
Area $= \pi r^2$
Circunferencia $= 2\pi r$

Sector circular

(θ en radianes)

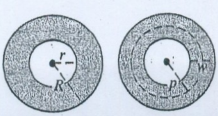
Area $= \frac{\theta r^2}{2}$

$s = r\theta$



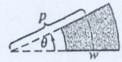
Anillo circular

(p = radio medio,
w = anchura)
Area $= \pi(R^2 - r^2)$
$= 2\pi p w$



Sector de un anillo circular

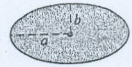
(p = radio medio,
w = anchura,
θ en radianes)
Area $= \theta p w$



Elipse

Area $= \pi ab$

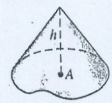
Circunferencia $\approx 2\pi \sqrt{\frac{a^2 + b^2}{2}}$



Cono

(A = área de la base)

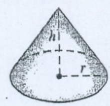
Volumen $= \frac{Ah}{3}$



Cono circular recto

Volumen $= \frac{\pi r^2 h}{3}$

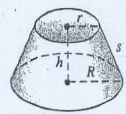
Area lateral $= \pi r \sqrt{r^2 + h^2}$



Tronco de cono

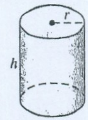
Volumen $= \frac{\pi(r^2 + rR + R^2)h}{3}$

Area lateral $= \pi s(R + r)$



Cilindro circular recto

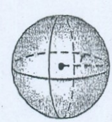
Volumen $= \pi r^2 h$
Area lateral $= 2\pi r h$



Esfera

Volumen $= \frac{4}{3}\pi r^3$

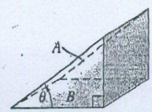
Area $= 4\pi r^2$



Cuña

(A = área de la cara superior,
B = área de la base)

$A = B \sec \theta$



ALGEBRA

Factores y ceros de polinomios

Sea $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ un polinomio. Si $p(a) = 0$, se dice que a es un *cero* del polinomio y una solución de la ecuación $p(x) = 0$. Además, $(x - a)$ es un *factor* del polinomio.

Teorema fundamental del álgebra

Un polinomio de grado n tiene n ceros (no necesariamente distintos). Aunque todos esos ceros pueden ser complejos, un polinomio real de grado impar ha de tener al menos un cero real.

Fórmula cuadrática

Si $p(x) = ax^2 + bx + c$, y $0 \leqslant b^2 - 4ac$, entonces los ceros reales de p son $x = (-b \pm \sqrt{b^2 - 4ac})/2a$.

Factorizaciones especiales

$x^2 - a^2 = (x - a)(x + a)$

$x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

$x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

$x^4 - a^4 = (x^2 - a^2)(x^2 + a^2)$

Teorema del binomio

$(x + y)^2 = x^2 + 2xy + y^2$

$(x - y)^2 = x^2 - 2xy + y^2$

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \cdots + nxy^{n-1} + y^n$$

$$(x - y)^n = x^n - nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 - \cdots \pm nxy^{n-1} \mp y^n$$

Teorema de los ceros racionales

Si $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ tiene coeficientes enteros, todos sus *ceros racionales* son de la forma $x = r/s$, donde r es un divisor de a_0 y s es un divisor de a_n.

Factorización por agrupamiento

$acx^3 + adx^2 + bcx + bd = ax^2(cx + d) + b(cx + d) = (ax^2 + b)(cx + d)$

Operaciones aritméticas

$ab + ac = a(b + c)$

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

$$\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \left(\frac{a}{b}\right)\left(\frac{d}{c}\right) = \frac{ad}{bc}$$

$$\frac{\left(\frac{a}{b}\right)}{c} = \frac{a}{bc}$$

$$\frac{a}{\left(\frac{b}{c}\right)} = \frac{ac}{b}$$

$$a\left(\frac{b}{c}\right) = \frac{ab}{c}$$

$$\frac{a - b}{c - d} = \frac{b - a}{d - c}$$

$$\frac{ab + ac}{a} = b + c$$

Exponentes y radicales

$a^0 = 1, \quad a \neq 0$

$(ab)^x = a^x b^x$

$a^x a^y = a^{x+y}$

$\sqrt{a} = a^{1/2}$

$\dfrac{a^x}{a^y} = a^{x-y}$

$\sqrt[n]{a} = a^{1/n}$

$\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$

$\sqrt[n]{a^m} = a^{m/n}$

$a^{-x} = \dfrac{1}{a^x}$

$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$

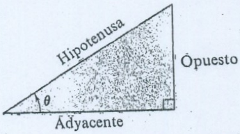
$(a^x)^y = a^{xy}$

$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

TRIGONOMETRIA

Definición de las seis funciones trigonométricas

Definición por triángulos rectángulos, con $0 < \theta < \pi/2$.

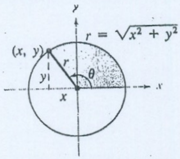


$$\text{sen }\theta = \frac{\text{op.}}{\text{hip.}} \qquad \text{cosec }\theta = \frac{\text{hip.}}{\text{op.}}$$

$$\cos\theta = \frac{\text{ady.}}{\text{hip.}} \qquad \sec\theta = \frac{\text{hip.}}{\text{ady.}}$$

$$\text{tg }\theta = \frac{\text{op.}}{\text{ady.}} \qquad \text{ctg }\theta = \frac{\text{ady.}}{\text{op.}}$$

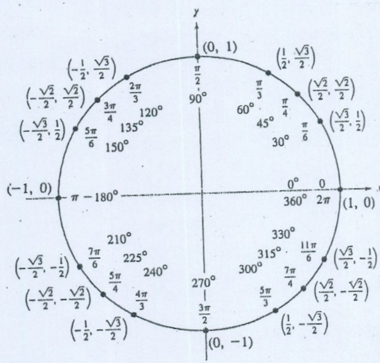
Definición como funciones circulares, para ángulos θ *arbitrarios.*



$$\text{sen }\theta = \frac{y}{r} \qquad \text{cosec }\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\text{tg }\theta = \frac{y}{x} \qquad \text{ctg }\theta = \frac{x}{y}$$



Identidades recíprocas

$$\text{sen } x = \frac{1}{\text{cosec } x} \qquad \sec x = \frac{1}{\cos x} \qquad \text{tg } x = \frac{1}{\text{ctg } x}$$

$$\text{cosec } x = \frac{1}{\text{sen } x} \qquad \cos x = \frac{1}{\sec x} \qquad \text{ctg } x = \frac{1}{\text{tg } x}$$

Identidades de tangente y cotangente

$$\text{tg } x = \frac{\text{sen } x}{\cos x} \qquad \text{ctg } x = \frac{\cos x}{\text{sen } x}$$

Identidades de Pitágoras

$$\text{sen}^2 x + \cos^2 x = 1$$

$$1 + \text{tg}^2 x = \sec^2 x \qquad 1 + \text{ctg}^2 x = \text{cosec}^2 x$$

Identidades de cofunciones

$$\text{sen}\left(\frac{\pi}{2} - x\right) = \cos x \qquad \cos\left(\frac{\pi}{2} - x\right) = \text{sen } x$$

$$\text{cosec}\left(\frac{\pi}{2} - x\right) = \sec x \qquad \text{tg}\left(\frac{\pi}{2} - x\right) = \text{ctg } x$$

$$\sec\left(\frac{\pi}{2} - x\right) = \text{cosec } x \qquad \text{ctg}\left(\frac{\pi}{2} - x\right) = \text{tg } x$$

Fórmulas de reducción

$$\text{sen}(-x) = -\text{sen } x \qquad \cos(-x) = \cos x$$

$$\text{cosec}(-x) = -\text{cosec } x \qquad \text{tg}(-x) = -\text{tg } x$$

$$\sec(-x) = \sec x \qquad \text{ctg}(-x) = -\text{ctg } x$$

Fórmulas de suma y diferencia

$$\text{sen}(u \pm v) = \text{sen } u \cos v \pm \cos u \text{ sen } v$$

$$\cos(u \pm v) = \cos u \cos v \mp \text{sen } u \text{ sen } v$$

$$\text{tg}(u \pm v) = \frac{\text{tg } u \pm \text{tg } v}{1 \mp \text{tg } u \text{ tg } v}$$

Fórmulas del ángulo doble

$$\text{sen } 2u = 2 \text{ sen } u \cos u$$

$$\cos 2u = \cos^2 u - \text{sen}^2 u = 2\cos^2 u - 1 = 1 - 2\,\text{sen}^2 u$$

$$\text{tg } 2u = \frac{2 \text{ tg } u}{1 - \text{tg}^2 u}$$

Fórmulas de reducción de potencias

$$\text{sen}^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\text{tg}^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Fórmulas suma-producto

$$\text{sen } u + \text{sen } v = 2 \text{ sen}\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\text{sen } u - \text{sen } v = 2 \cos\left(\frac{u + v}{2}\right)\text{sen}\left(\frac{u - v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u + v}{2}\right)\cos\left(\frac{u - v}{2}\right)$$

$$\cos u - \cos v = -2 \text{ sen}\left(\frac{u + v}{2}\right)\text{sen}\left(\frac{u - v}{2}\right)$$

Fórmulas producto-suma

$$\text{sen } u \text{ sen } v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos u \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\text{sen } u \cos v = \frac{1}{2}[\text{sen}(u + v) + \text{sen}(u - v)]$$

$$\cos u \text{ sen } v = \frac{1}{2}[\text{sen}(u + v) - \text{sen}(u - v)]$$