Complexity and Stop Conditions for NP as General Assignment Problems, the Travel Salesman Problem in $\mathbb{R}^2$, Knight Tour Problem and Boolean Satisfiability Problem

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Abstract

This paper presents stop conditions for solving General Assignment Problems (GAP), in particular for Travel Salesman Problem in an Euclidian 2D space the well known condition Jordan's simple curve and opposite condition for the Knight Tour Problem. The Jordan's simple curve condition means that a optimal trajectory must be simple curve, i.e., without crossing but for Knight Tour Problem we use the contrary, the factible trajectory must have crossing in all cities of the tour. The paper presents the algorithms, examples and some results come from Concorde's Home page. Several problem are studied to depict their properties. A classical decision problem SAT is studied in detail in order to state that NP is not P.

Keywords: Algorithms, TSP, NP, Numerical optimization.

1 Introduction

I invite everybody to read this paper for discuss and interchange knowledge.

Build software is similar to prove theorems, i.e., to study, think, plan, revise, cut, change, understand in order to pursuing an esthetic beauty for an efficiency procedure for solving the questions or problems. I prefer constructivism on mathematical model, properties, theories over heuristic or recipes. So, lacking of a formal proof the dispute of P $\neq$ NP will continue. Or it could be settle, by a group willing to interchange ideas, fields and open their minds. This is no new, it is happen with physicians, they are more open in the pursuing of the understanding of the realm of science and universe. Other answer is the lack of people on the field of Computer Science with interest on the promise of one million of dollar from the Millennium Problems of Clay Mathematics Institute, perhaps they has no time to investigate after Cook, et al. [18]. I believe that in México and in the world there are a few people include myself dedicated to explore difficult topics of Science. Many universities have no department on Computer Science. My next article for reviewing languages, automates and computation models for depicting that nowadays there is not exist such thing as an artificial intelligent being but the human being. My opinion and claim for the world leaders, politicians, and education state secretaries is give support around the world to educate on basic sciences and culture to all the people.

The NP problems [18], the 2D Euclidian Traveller Salesman (hereafter, TSP) has been studied abroad. The Combinatorics and Optimization Department of the University of Waterloo has made an splendid research and offer in my opinion the best free software for TSP: the Concorde software [1].

This paper complement my previous papers [10, 11], they have mistakes but ideas that let me advance in the study of open statement: NP is not P. I take a while to work and continue to prove it.

When I wrote [10], I was not able to state more properties of the TSP and some result of polinomial complexity for triangulation [3] make me hesitate that TSP could have polinomial time algorithm in $\mathbb{R}^m$, $m \geq 3$. Here, I

present the correction of my mistake with a carefully study of the properties of similar problems TSP, Knight tours problem (KTP), polygonals (as TSP type), and Boolean Satisfiability Problem (SAT).

For TSP and KTP, there are abroad articles and publications, I mentions two for my preference, of course other are quite important: [4, 19, 20].

The preliminary idea and research was presented at the XVIII Workshop of Mathematical Analysis and Applications at November 14, 2012. In fact, there was a dispute with one college (who studied his PHD at the University of Waterloo). He challenge me that the idea of using Jordan's simple curve was not right to development an efficient algorithm. The Jordan's simple curve come from [4] figures 1.21 and 1.22 (pag. 24) as uncrossing the tour segments.

I did an invitation of UAM's students to transform the Concorde Software to the USS Enterprise [8] by incorporating an adaptation in the Concorde Software [1] to my idea but no student replies to me. Here, I present the results from an ad-hoc Matlab programs that could interesting to development a fully automatic program for TSP.

To my knowledge, what makes different my novel approach (using Jordan's simple curve) is the combination of visualization data techniques with a very simple optimization algorithms using greedy and uniform search strategies and the study of mathematical properties for understanding stop conditions and when they are necessary or sufficient. This means that optimization program are polinomial time when the search space for TSP is convex but yes-no program for GAP_n in general could have a large research space without an appropriate data organization, therefore there is not polinomial time algorithm for answering them.

In general, for the TSP under euclidian distance (or where the coseno law states) Jordan's simple curve is a necessary condition on a plane but sufficient. This is similar to solve by the necessary condition $f'(x) = 0$, an global optimization problem with objective function f, which is derivable. This is the well known first order necessary condition in optimization. It does not imply the optimality. To be sufficient it is needed, by example, that f must be quadratic or convex. For the TSP, Jordan's simple curve condition provides a stop condition (necessary condition, see Prop. 4.3) to build an approximation to the solution and with other geometrical properties as convexity, it becomes a necessary and sufficient condition (see Prop. 4.4).

Now, I can state a similarity between to solve a NP Hard with global optimization problems, where in the last one, it is well known that there is not an general and efficient algorithm to solve any global optimization problem. Also, my approach is to have a visual property or a very clever indicator to allow us the corroboration of the solution without additional computational cost. This is the property that could be used to the efficient verification of a NP given instance.

I will try to avoid any dispute between mathematicians but it is difficult to satisfice them. Hopping, I present proofs that the discrete mathematicians, applied mathematician, pure mathematician, logicians and computer scientist could to be agreed. They do not like to mix their subjects, methods and approaches. Also, my colleges of my department do not understand how to solve the next optimal control problem. I got a rejection letter for trying to give a talk at the XX Anniversary of Mathematical Analysis and Its Applications. Their letter shows that they do not know about Control Theory over PDE, and I think that they do not like anything outside of their safety mathematical comfort zone.

Briefly, an approach for solving an Optimal Control Problem over a Controlled System of Partial Differential Equation consists: to have a state a quadratic variacional objective function $J : \mathcal{V} \to \mathbb{R}$, by example:

$$J(v) \;=\; \frac{C_0}{2} \sum_{k=0}^{M} \int_0^T v_k^2 \mathrm{d}t + \frac{C_1}{2} \iint_Q y^2 \mathrm{d}x \mathrm{d}t + \frac{C_2}{2} \int_0^L y(x,T)^2 \, \mathrm{d}x$$

where $\mathcal{V}$ is an appropriate Hilbert Space of integrable functions with $\langle u, v \rangle = \int_0^T uv \mathrm{d}t$, $C_0, C_1, C_2 > 0$ are given constants, $v_k \in \mathcal{V}$, $x_k \in [0, L]$ are the control points, $x_k = L(k/M), k = 0, \ldots, M$, $\mathbf{0}$ is the steady final state at $t = T$, and y is the solution of the following system of controlled partial differential equations:

$$\begin{cases} \frac{\partial y}{\partial t} - \mu \frac{\partial^2 y}{\partial x^2} + \epsilon \frac{\partial y}{\partial x} - y = \chi_{x_k}(x) v_k & \text{in } Q = (0, L) \times (0, T), k = 1, \ldots, M-1 \\ y(x, 0) = y_0, & t = 0, \\ -\mu \frac{\partial y(0,t)}{\partial x} = v_0, & x = 0, \\ \mu \frac{\partial y(L,t)}{\partial x} = v_M, & x = L. \end{cases} \tag{CSE}$$

2

The optimization problem is:

$$\min_{v \in \mathcal{V}} J(v) = \frac{C_0}{2} \sum_{k=0}^{M} \int_0^T v_k^2 \mathrm{d}t + \frac{C_1}{2} \iint_Q y^2 \mathrm{d}x \mathrm{d}t + \frac{C_2}{2} \int_0^L y(x,T)^2 \, \mathrm{d}x, \tag{OP}$$

where y is the solution of (CSE) for v. Note that the state research space of function is not empty. For example, the steady state $y(x) = \mathbf{0}$, and $v = \mathbf{0}$ correspond the state equations system:

$$\begin{cases} \frac{\partial y}{\partial t} - \mu \frac{\partial^2 y}{\partial x^2} + \epsilon \frac{\partial y}{\partial x} - y = 0, & \text{in } Q = (0,L) \times (0,T), \\ y(x,0) = 0, & t = 0, \\ -\mu \frac{\partial y(0,t)}{\partial x} = 0, & x = 0, \\ \mu \frac{\partial y(L,t)}{\partial x} = 0, & x = L. \end{cases} \tag{SE}$$

Under the appropriate integral-differential conditions, the optimization problem (OP) is soluble. The technique of the solution is based on the estimation of the first variation of J, i.e., this is similar to use the first derivative condition which is a necessary and sufficient condition for a quadratic objective function. Also, the control v is a minimum. It corresponds to the minimum cost of the control, also, J keeps low the cost of the evolution of the system $y(x,t)$, and the cost of final state when is closed to $z(x)$. Here, there are mathematical properties, and a control law to guaranty using, by example, the conjugate gradient method, how to get an approximation of the solution, i.e., it exists a mathematical (analytically) controlled law (see [12, 15]).

I receive a kind personal letter from Rustem Valeyev to read his article [14]. He states, in my opinion, that in any vertices that there are not a clear path to follow, I quote his phrase "*absence of analytically controlled law,"irregularity")*". The meaning is similar to my statement "*large GAPn, has not an inherited property allowing to build an efficient algorithm for verifying the solution, moreover, to solving it*" from the conclusions of [10]. Thanks to Mr. Rustem in his last reply: "I thank you for attention and goodwill. I shall be glad to answer any your questions if they will arise. Also I do not think myself the owner of true in last instance :)", P $\neq$ NP.

Finally, I will refer to the reader to my previos papers [10, 11] to avoid repeat notation, propositions, and proofs (They have mistakes, I am working to correct them). However, I will include some parts to make this paper self-explicatory. Finally, I shall be glad to answer any your questions by email.

2   Travel Salesman Problem, and Knight Tour Problem like General Assign Problem

The well known Travel Salesman Problem (TSP) and Knight Tour Problem (KTP) are particular cases of the General Assign Problem(GAP), some properties and characteristics are [10].

The GAP_n consists a complete graph, a function which assigns a value for each edge, and objective function, $\text{GAP\_}n = (G_n, c, f)$, where $G_n = (V_n, A)$, $V_n \subset \mathbb{N}$ $A = \{(i,j) \mid i,j \in V\}$, $c : V \times V \to \overline{\overline{\mathbb{R}}}$, and f is a real function for the evaluation of any path of vertices.

In particular, without lost of generality, I focus in minimization to solving a GAP. It means to look for a minimum value of f over all hamiltonian cycles of G_n. Also, it is trivial to see that any GAP_n has a solution.

The main proposition in [10], it depicts contradictory properties and characteristics for solving GAP_n:

Proposition 6.12. An arbitrary and large GAP_n of the NP-Class has not a polynomial algorithm for checking their solution.

In this article, $\text{SAT}_{n \times m}$ is used to state:

Proposition 9.10. $\text{SAT}_{n \times m}$ has not property or heuristic to build an efficient algorithm.

Proposition 9.11. NP has not property or heuristic to build an efficient algorithm.

In my previous articles, I did not give an explication of why I selected graph as the main core of my study. From the [13]: "Euler called this new mathematics without numbers, in which only the structure of the configuration plays a role, but not size and form, Geometria situs, geometry of position (he took the concept from a letter of Leibniz from the year 1679)." Methods for solving the NP look for a common, and global property but using

graphs, they depict local properties, i.e., properties related to immediately vicinity of the nodes. Therefore, what is valid in a situs could be invalid in other positions. Here, on the other hand, a simple tipo of SAT_n is studied to support that there is not a property to build an efficient algorithm to avoid an exhaustive searching of the search space or to let to build knowledge from a polinomial population of cases.

The travel salesman problem with cities as points in a m-dimensional euclidian space (TSP_n) is a special case of a GAP_n where the edge's cost is given by the matrix $\mathcal{C} = [c(i,j)], c(i,i) = \infty, i = 1, \ldots, n, j = 1, \ldots, n$. $c(i,j) = $ euclidian distance(i,j) $\forall i \neq j \in V_n$, and the vertex i is a point in $\mathbb{R}^m, m \geq 2$. The elements of the diagonal of $\mathcal{C}$ are equals to ∞, this is to avoid getting stuck. TSP2D (Hereafter, TSP) is a problem with cities as points in $\mathbb{R}^2$ and edges's values come from their euclidian distances between cities.

In similar way, the Knight tour problem (hereafter, KTP) is an special special case of a GAP_n where the vertices are fixed in each square's center of a chessboard $n \times m$. The squares'coordinate are given by their integer coordinates of the chessboard's squares. The edge's cost is given by the matrix $C = [c(i,j)], c(i,i) = \infty,$ Euler distance(i,j) $\forall i \neq j$. To honor the great Mathematician Leonard Euler, I called it as Euler's distance (see [20]).

$$\text{Euler's distance}(i,j) = \begin{cases} c_1 & \text{if } (i-i')^2 + (j-j')^2 = 5 \\ \infty & i = j \\ \sqrt{\left((i-i')^2 + (j-j')^2\right)} + 4 & \text{otherwise} \end{cases}.$$

My first approach was select $c_1 = 0.01$. However, in order to privilege the knight moves, c_1 needs a more carefully tuning (see section 5). The problem consists to look for a hamiltonian cycle in a chessboard of 8×8 with cost less than 4.

The [19] depicts in chapters 40 through 42 results about uncrossed, celtic, long and skinny knight's tours.

The following propositions are known, I include for clarity.

Proposition 2.1. *Any TSP is soluble, i.e. there is a minimum cost hamiltonian cycle.*

Proof. It obvious, a TSP has n cities, and its a complete graph. Then by comparing the all (n-1)! hamiltonian cycles the solution is found. □

Proposition 2.2. *A graph knight is a graph $m \times n$ squares if only if $\forall mn$ vertices (i,j), (i',j') with $1 \leq i < i' \leq m$, and $1 \leq j < j' \leq n$, the square distance, $(i-i')^2 + (j-j')^2 = 5$.*

Proof. See [19]. A knight walks in a chessboard in L form from a white square to a black square. In order, for a knight moving around all vertices, they need to be adjacent vertices to allowable the motion of the knight. □

Remark 2.3. For the TSP and KTP we are interesting to find a minimum hamiltonian cycle.

However, the previous proposition does not considere exactly KTP, a hamiltonian cycle with only knight´s move but hamiltonian path.

The idea of the Euler distances function come from the previous proposition to provide space and to favor the moves of a knight. Some results from [19] can be verified by doing exhaustive search. I create a a simple Matlab programs to verify some knight's tour in small chessboards (if you want a copy, email me).

From the last two propositions, I define what type solutions to focus in this paper:

1. for the TSP, a minimum cost hamiltonian cycle, and

2. for the KTP a crossing hamiltonian cycle with only knight´s move.

The TSP correspond to calculate and prove the optimality and for the KTP is to find a special hamiltonian cycle.

They are quite different. Nevertheless, for any objective function f of a GAP_n, hamiltonian's path or cycle are computable problems and the minimum selection procedure could be used to find the solution (see proposition 3.2 in [10]). The minimum selection procedure in mathematical notation is:

$$y^* = \arg \min_{y = (v_1, v_2, \ldots, v_n, v_1), v_1 \in V_n, v_i \in V_n, v_i \neq v_j, 1 \leq i < j \leq n} \{f(y) \mid \text{GAP}_n = (G_n, c, f), G_n = (V_n, A)\} \qquad (2.1)$$

It is clear that both TSP and KTP are GAP'_ns type with a similar type of an objective function, f, which is the summation of the edges' cost over the consecutive pairs of vertices of a path. However, TSP is a hard NP

problem, similar to a global optimization problem. On the other hand, KTP is a decisión problem to look for a crossing hamiltonian cycle.

The Research Space of GAP_n is finite and numerable, and it has $(n-1)!$ elements (see Prop.3.4 [10]). It is totally impractical, to solve eq. 2.1 by an exhaustive searching procedure.

3   A greedy algorithm for Eq. 2.1

Algorithms for solving global optimization problems have abundant literature. For arbitrary objective functions on a bounded subset, $B \subset \mathbb{R}^m$, uniform searching combined with local optimization can be used to estimate the global optimal solution (see Theorems of Convergence: Global and Local Search in [21]). The convergence to the solution has not guaranty of polynomial time complexity and the solution can be found at the cost of thoroughly searching on B. This motivate to create heuristic methods, by example, I worked in heuristic methods:

1. Classical and Exponential Tunnelling method [5, 16, 17]. These methods do not guaranty to find the global solution but a descend of the objective function's value. The user defines and decides the number of iterations or the time's execution.

2. Genetic Algorithms for Global Optimization for the LJ Problem [6, 7, 9].

The following heuristic greedy algorithm execute an uniform search on the vertices, or select the closed vertex, and repeat this procedure K times ($K >> 0$):

Algorithm 1. **Input:** GAP_n with $\mathcal{C} = [c(i,j)]$, matrix of the edge's cost, a hamiltonian cycle, $V_o = (v_1, v_2, \ldots, v_n, v_1)$, with minimum cost c_o.

Output: $v_o = (v_1, v_2, \ldots, v_n, v_1)$ a hamiltonian cycle with minimum cost c_o.

1. **For** $r = 1$ to K

2.   **Select randomly an initial vertex by column or row** i_v, **or** j_v **from** $1 \leq i_v, j_v \leq n$.

3. $j_v = \arg\min_{1 \leq j \leq n} [c(i_v, j)]$ **or** $i_v = \arg\min_{1 \leq i \leq n} [c(i, j_v)]$;

4. $v_1 = i_v$ or j_v;

5.   **end select**

6. $i = 1$;

7. $V_a = (v_1)$.

8.   **While** V_a **is not a hamiltonian path**

9. $v_{i+1} = $ **Select randomly** $\left\{ \begin{array}{l} \arg\min_{1 \leq j \leq n} [c(v_i, j)] \\ \textbf{Selectrandomly } v_j \notin v_a \end{array} \right.$;

10.   **if** $v_{i+1} \notin V_a$ **then**

11.   **add** v_{i+1} **to** V_a;

12.   **else**

13. $i = mod(i+1, n) + 1$;

14.   **end if**

15.   **end while**

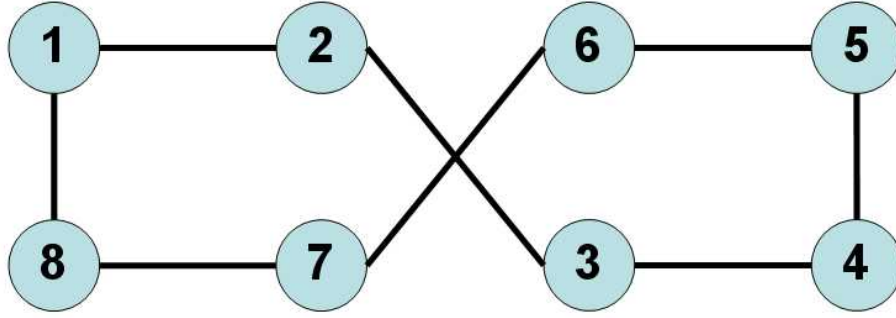16. $V_a = (v_1, v_2, \ldots, v_n, v_1)$;

Figure 1: Hamiltonian cycle with a crossing in the cities: v_2, v_3, v_6, v_7.

17. $c_a = f(V_a)$;

18.   **if** $c_a < c_o$ **then**

19. $V_o = V_a$;

20. $c_o = c_a$;

21.   **end if**

22. **end for** r

Remark 3.1. A putative minimum hamiltonian cycle can be obtained by the previous algorithm. Or it keeps the current putative minimum hamiltonian cycle. The value of K is 200.

To behave only greedy the step 9 could be changed to $v_{i+1} = \arg min_{1 \le j \le n} [c(v_i, j)]$. And reciprocally, to set on uniform search this step could be changed to $v_{i+1} = $ **Select randomly** $v_j \notin v_a$

4   Algorithm for TSP

Prop. 6.11 in [10] depicts that for any quadrilateral, its sides are lower than its diagonal. This section depicts algorithms for solving TSP using this property.

Algorithm 5 in [10] is used to order the vertices according to a given hamiltonian cycle. Hereafter, we assume that the current hamiltonian cycle is in ascending order of the consecutive cities. It is not necessary, but assuming that the vertices of the hamiltonian cycle are in order facilitate the description of the following algorithms.

Let's assume that with algorithm 1, there is a hamiltonian cycle for a given TSP. An image of the current solution can be obtained by a graphic interface as in Concorde or using the graphic's tool of Matlab or Octave. By example see fig. 1.

Algorithm 2. **Input:** An black and white image of the current hamiltonian cycle, $v = (v_1, v_2, \ldots, v_n, v_1)$ where the cities are in order consecutive.

Output: Stop, the image of the hamiltonian cycle can be colored by two colors. Otherwise, the hamiltonian cycle has a crossing at the cities $v_i, v_{i+1}, v_j, v_{j+1}$.

1. **Using** v_1 detects its location in the input image.

2. **Using** v_1 detects its frontier in the image for selecting two points, one inside and one outside of the v_1.

3. **Using** a flood or a paint graphic tool with the point outside of the v_1, colored the image on green.
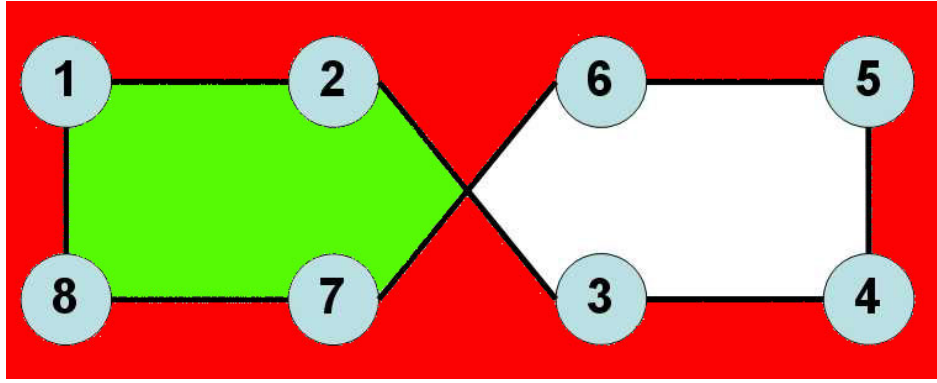
6

Figure 2: Colored image of a hamiltonian cycle with a crossing in the cities: v_2, v_3, v_6, v_7.

4. **Using** a flood or a paint graphic tool with the point inside of the v_1, colored the image of red.

5. two_color="yes";

6. **for** $i := 1$ **to** n

7.   **Using** v_i detects its location in the colored input image;

8.   **if** the vicinity of the v_i has red, green and black **then**

9.   **continue**;

10.   **else**

11.   two_color="no";

12.   **mark** v_i;

13. **end for** i;

14. **if** two_color is "yes" **then**

15.   **Stop.** "The image of the hamiltonian cycle can be colored by two colors,$v = (v_1, v_2, \ldots, v_n, v_1)$ is a hamiltonian cycle as Jordan's simple curve, i.e., without crossing.";

16. **select** marked vertex, let assume it is v_i

17. **using** v_i detects its location in the colored input image;

18. **using** v_i detects its four closed marked neighbors;

19. **Stop.** There is crossing at the cities: $v_i, v_{i+1}, v_j, v_{j+1}$.

Remark 4.1. Flood or paint graphic tool algorithm has the complexity of the image's size is $\mathbf{O}(kn^2)$ where kn is related to the resolution of the vicinity of the n vertices. The value of factor k depends of the minimum distance of the vertices. k must be allow to have a black-white image where the lines of hamiltonian cycle are clearly distinguish. Figure 2 depicts when there is a crossing.

Assuming that vertices of the hamiltonian cycle are in order, the only one solution for figure 2 is depicted in figure 4. Figure 3 depicts the wrong connection. There are two possibilities when a crossing is found, but only one is the correct on the current hamiltonian cycle. Nevertheless, the cost of the cycle always decreases. Therefore, if there is crossing at the cities $v_i, v_{i+1}, v_j, v_{j+1}$ the hamiltonian cycle must connects v_i to v_j and v_{i+1} to v_{j+1} and there is always a guaranty for decreasing cycle's cost from Prop. 6.11 in [10].

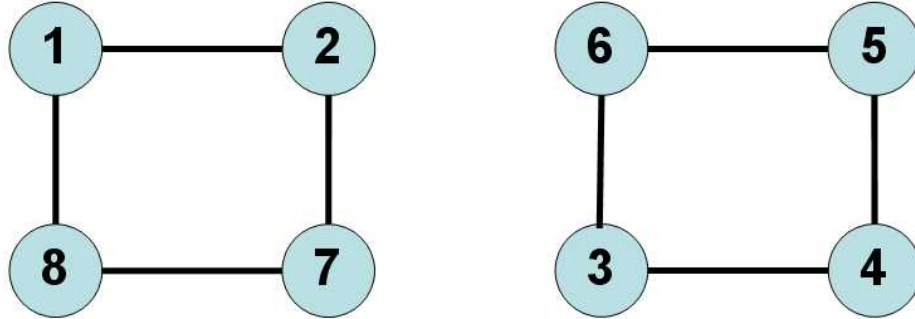Without loss of generality, let's assume that $i < j$.

Figure 3: Wrong connection of crossing in the cities: v_2, v_3, v_6, v_7.
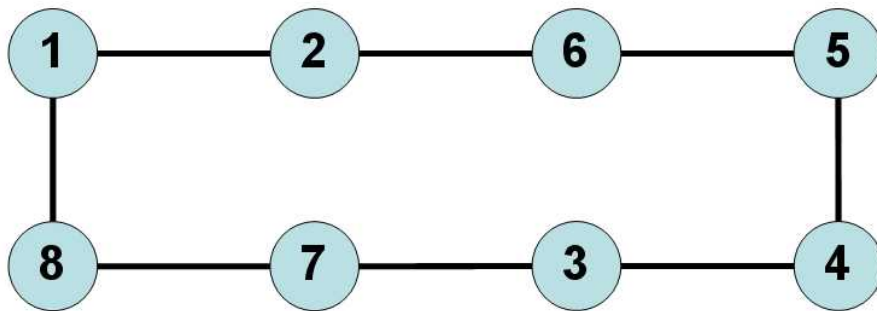

Figure 4: Hamiltonian cycle without the crossing in the cities: v_2, v_3, v_6, v_7.

Algorithm 3. **Input:** $V = (v_1, v_2, \ldots, v_n, v_1)$ a hamiltonian cycle, where the cities are in order consecutive and with a crossing in four cities: $v_i, v_{i+1}, v_j, v_{j+1}$.

    Output: $V = (v_1, v_2, \ldots, v_n)$ a hamiltonian cycle without the crossing at the cities $v_i, v_{i+1}, v_j, v_{j+1}$.

1. $V' = V$;

2. $l = j$

3. **for** k:= i+1 **to** j

4. $V(k) = V'(l)$

5. $l = l - 1$;

6. **end for** k;

7. **stop.** The hamiltonian cycle without the crossing is $V = (v_1, v_2, \ldots, v_i, v_j, v_{j-1}, \ldots, v_{i+1}, v_{j+1}, \ldots, v_n, v_1)$.

Remark 4.2. The complexity of the previous algorithm is $\mathbf{O}(n)$. The next steps are for ordering the cities of the hamiltonian cycle, perhaps to repeat a greedy algorithm for looking a less hamiltonian cycle's cost around the current solution. The next paragraph depicts the complete algorithm for solving the TSP, using the previous algorithms to get a simple Jordan's simple curve. This is a necessary condition to stop, because the crossing of the cities is a visual property for detecting where the hamiltonian cycle's cost decreases.

Algorithm 4. **Input:** TSP.

    Output: $V = (v_1, v_2, \ldots, v_n, v_1)$ a hamiltonian cycle as Jordan's simple curve, i.e., without any crossing on the path of cities.

1. **Repeat**

2.   **execute algorithm** 1;

3.   **execute algorithm 5** in [10] to order the hamiltonian cycle by their consecutive cities;

4.   **create a black-white image**;

5.   **execute algorithm** 2

6. **until get a Jordan's simple curve from the current hamiltonian cycle.**

7. **Stop.** The hamiltonian cycle is the putative solution of the TSP.

Proposition 4.3. *Given a TSP_n, where cities are points in $\mathbb{R}^2$, and the cost matrix correspond to euclidian distances between cities. The Jordan's simple curve is a necessary condition for the minimum hamiltonian cycle's cost.*

Proof. With out loss of generality, if the putative optimal hamiltonian cycle has a crossing then for Prop. 6.11 in [10], the hamiltonian cycle obtained by algorithm 3 has lower value than the putative optimal hamiltonian cycle. $\qquad\square$

Proposition 4.4. *Given a TSP_n, if cities are located around a closed convex curve, the algorithm 4 finds the hamiltonian cycle of minimum cost.*

Proof. The optimality of the hamiltonian cycle comes from the vertices'configuration around of convex curve, points on a circle, elipse, n-poligonal. The algorithm 1 by construction selects the closed next vertex. The hamiltonian cycle corresponds to the solution of the variational problem of the the minimum lengths curve (Jordan's simple curve) with the maximum convex area. $\qquad\square$
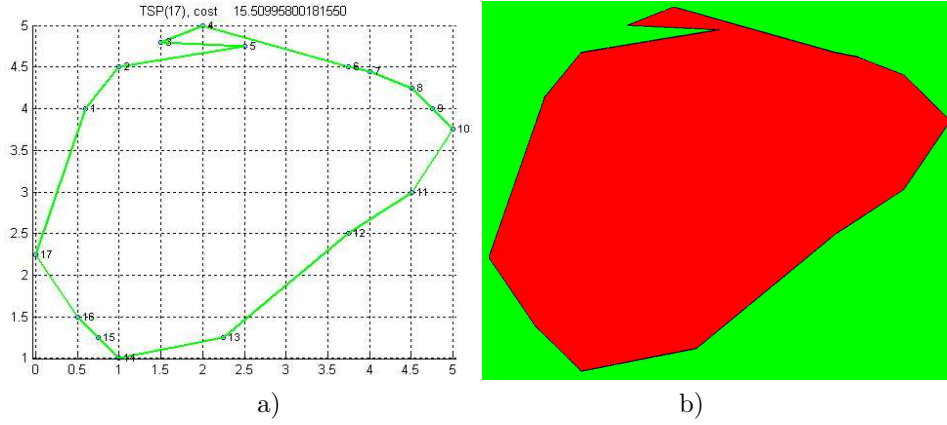
Figure 5: TSP$_{17}$. a) Not optimal hamiltonian cycle with Jordan's simple curve, and b) its two colored image.
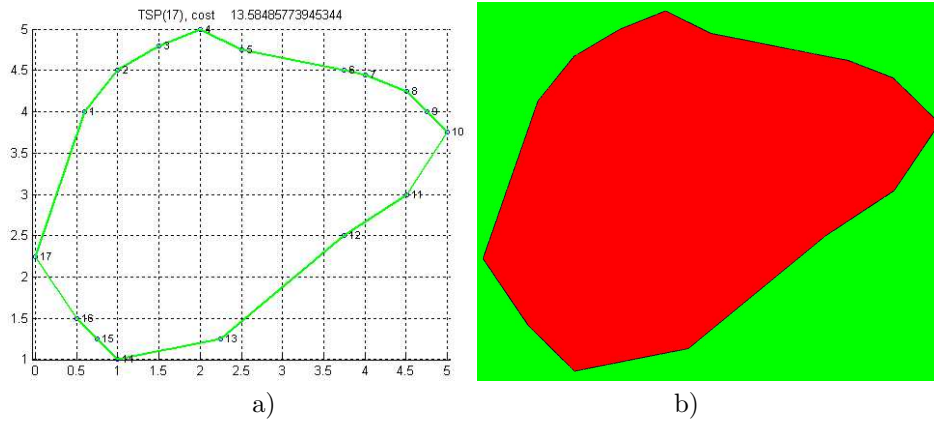

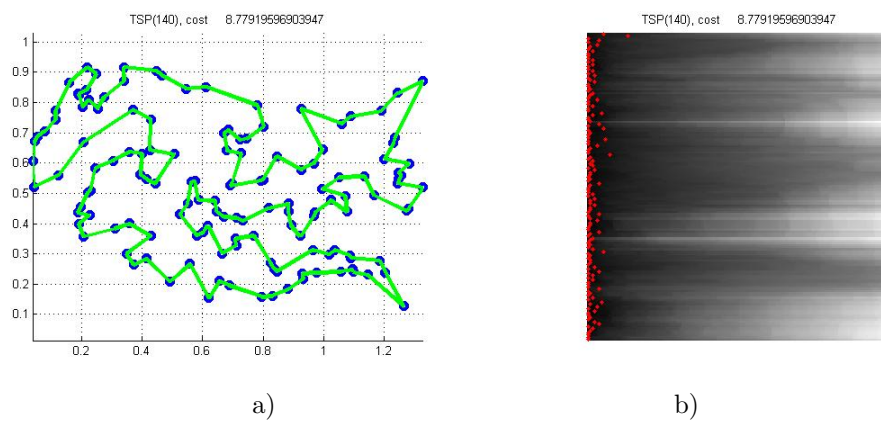Figure 6: TSP$_{17}$. a) Optimal hamiltonian cycle with Jordan's simple curve, and b) its two colored image.

Figure 7: TSP$_{140}$. a) Hamiltonian cycle with the minimum cost, and b) sorted cost matrix $\mathcal{M}$. The red dots on the left side implies that there are not many alternatives hamiltonian cycles.

Remark 4.5. Figure 5 depicts a case where the algorithm 2 fails to detect the zone where it is possible to descend the cost. However, there are many algorithms where this not happen. I hope, that the algorithms of the Concorde [1] for TSP2D get advantage of the Jordan's simple curve property in the future.

Unfortunately, for TSP where the objective is to get a minimum hamiltonian cycle's cost related to money, time travel or arbitrary values then Jordan's simple curve property is not useful. This includes the max distance giving by: $d_m(x,y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$, where $x = (x_1, x_2)$, and $y = (y_1, y_2) \in \mathbb{R}^2$. It is easy to verify that a quadrilateral under d_m has diagonals not larger than its sides.

Figures 5 and 6 depict a case where the previous proposition is verified for a small TSP_{17}. Red area of the former figure has $415, 570$ pixels with cost $= 15.5100$, and red area of next figure has $417, 719$ pixels with cost $= 13.5849$. Also, these figures shows why the Jordan's simple curve is a necessary property.

Using algorithm 4, big TSP can be solved to a putative optimal hamiltonian cycle. Figure 7 taken from [10] (Figure 16) depicts a Jordan's simple curve. The red dots depicts the minimum hamiltonian cycle, it corresponds to vertices' enumeration closed to the left side. In this case the complexity of the algorithm 4 until getting a Jordan's simple curve is polinomial time bounded by $\mathbf{O}(n^k)$ with k a small positive integer, and the time depends of the many crossings that the greedy algorithm 1 can to avoid.

5 Algorithm for KTP

The Euler's distance of the KTP does not connect the squares of a chessboard as usual, but it privilege the knight's moves. It is clear that Prop. 6.11 in [10] (sides versus diagonals of quadrilaterals) can not be used for solving KTP, i.e., the algorithm for the KTP is looking for the contrary of a simple Jordan's simple curve as in TSP, in fact, the property is a hamiltonian cycle with many crossing as possible.

The algorithm 1 is used without a modification. But, here the goal is to stop when the cost of the hamiltonian cycle is less than 4.

To my knowledge there is not a proposition or theorem for proving that exists a complete crossing knight tour for any chessboard's size. In [2] there is a table of solutions for some chessboard's size. I tune the Euler's distance function using the description of the article [20]. The knight's motion has three different behaviors each quadrant of size 4×4:

1. Two romboides with direction $(-1, -1)$ to $(1, 1)$;

2. Two romboides with direction $(-1, 1)$ to $(1, -1)$;

3. Two squares.

Algorithm 5. **Input:** KTP for a chessboard of 8×8.
   Output: $V = (v_1, v_2, \ldots, v_n, v_1)$ a hamiltonian cycle with cost less than 4.

1. **Execute algorithm** 1.

2. **while current hamiltonian cycle's cost $\geq$ 4 do**

3.   **execute algorithm 5** in [10] to order the hamiltonian cycle by their consecutive vertices;

4.   **execute algorithm** 1;

5. **end while.**

6. **Stop.** The hamiltonian cycle is the solution of the KTP.

There is not guaranty that the previous algorithm is computable, i.e., it could not to stop for an arbitrary chessboard's size. I mean that the tuning of c_1 works for a chessboard of 8×8. For arbitrary chessboard's size, it could not provide a hamiltonian path because the corresponding GAP could not have a hamiltonian cycle with cost less than 4.

The selection of the value 4 is in the hope that the greedy part of the algorithm 1's behaves greedy when it searches for minimum hamiltonian cycle's cost. It will prefer to choice edge's cost between vertices that
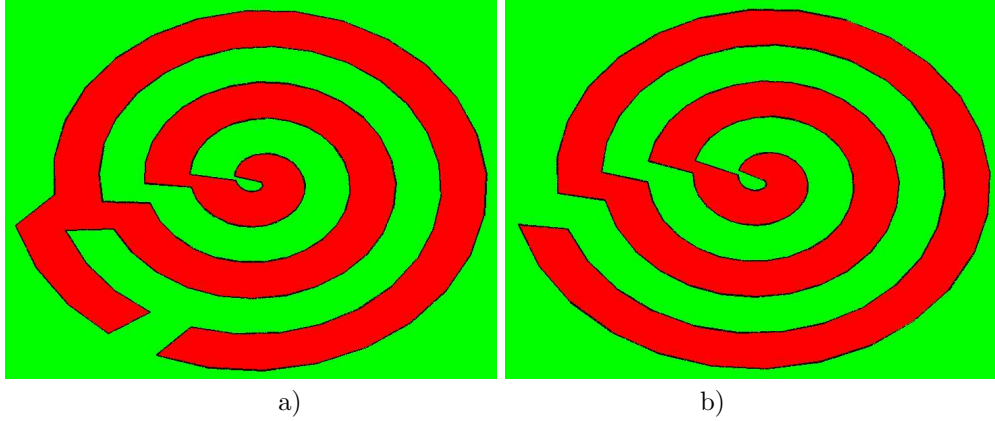
Figure 8: TSP_{152} on a spiral. Two colored Jordan's simple curve: a) not optimal hamiltonian cycle 21.1661, and b) optimal hamiltonian cycle 21.1451.
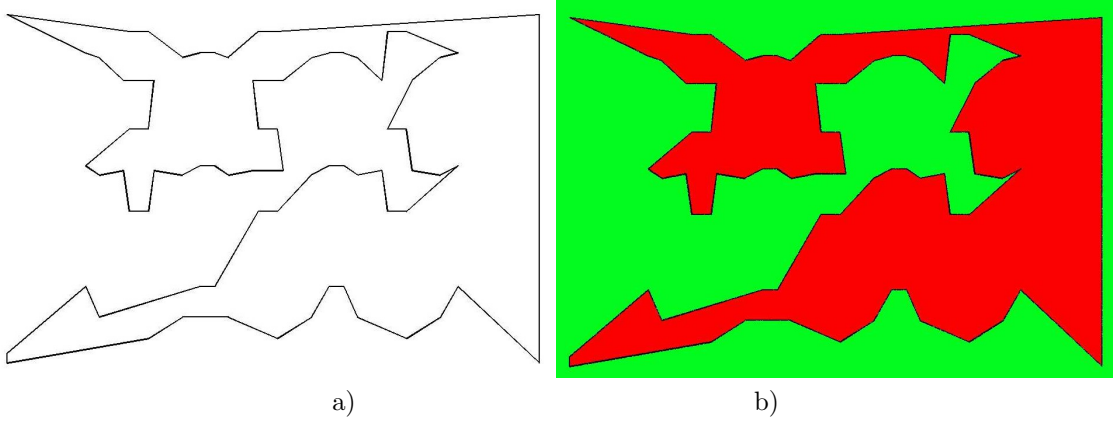

Figure 9: TSP_{76}(file pr76.tsp). a) final hamiltonian cycle with Jordan's simple curve, and b) its two colored image.

corresponds knight's paths. By construction the total distance of a tour with only knight's motion is less than 4. For any pair of squares that not correspond to knight's paths the distance is giving by $\text{sqrt}\left((i - i')^2 + (j - j')^2\right) + 4$, .i.e. when $(i - i')^2 + (j - j')^2 \neq 5$ where (i, j) and (i', j') are the integer coordinates of the squares.

For a knight's paths the value of c_1 of the Euler's distance is 0.04, but in each quadrant of size 4×4 the value of c_1 is:

1. 0.01 for the two romboides with direction $(-1, -1)$ to $(1, 1)$;

2. 0.03 for the two romboides with direction $(-1, 1)$ to $(1, -1)$;

3. 0.02 for the two squares.

Remark 5.1. The heuristic idea to assign c_1 values come from the article [20], and without any formal proof, it works for KTP 8×8 using the $\text{GAP\_}n$'s algorithm 1.

6   Numerical Experiments for TSP and KTP

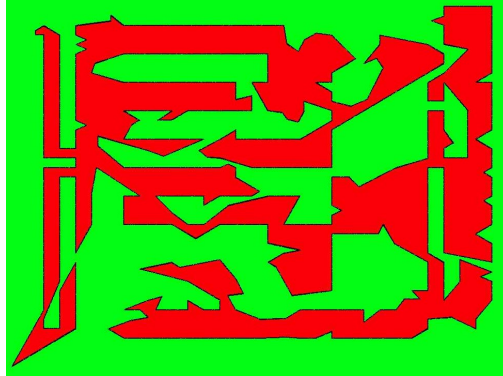Figure 10: TSP$_{442}$(file pcb442.tsp). Final two colored hamiltonian cycle with cost $= 56,601.1738$.
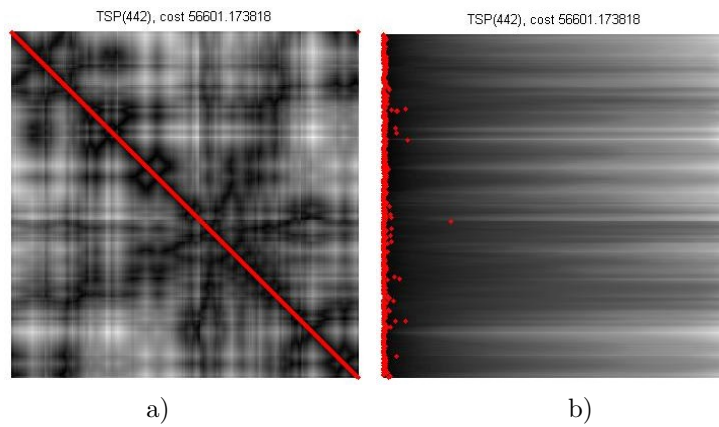

Figure 11: TSP$_{442}$ final hamiltonian cycle. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.
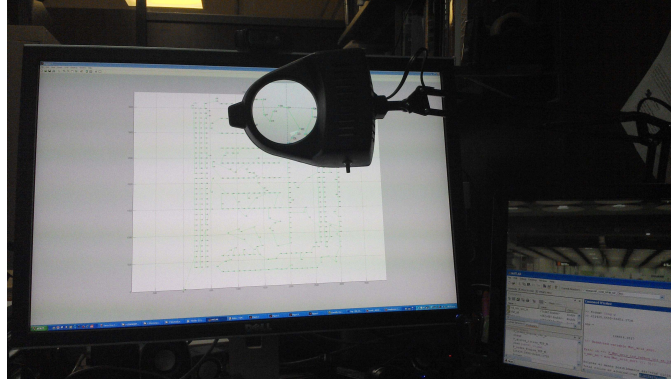
Figure 12: Computer Equipment.

The first numerical experiment is for TSP with 152 cities located on a spiral curve. TPS_{152}. Figure 8 depicts two very closed hamiltonian cycles. On a) is the not optimal, its red area has $161,279$ pixels, and on b) is the putative optimal, its red area has $158,813$ pixels. The cost's difference between the two hamiltonian cycles is only 0.021. Also, the cities locations are not fulfil the hypothesis of proposition 4.4, therefore their red areas'size are not as in figures 5 and 6.

From [1] the files pr76.tsp and pcb442.tsp provide the results depicted on figures 9 and 10. The putative optimal complex hamiltonian cycle of the TSP_{442} is not the global optimal. Figure 11 depicts on b) many red dots far away of the left side, as I explains in [10] this means that exists yet many alternatives hamiltonian cycles to explore. A rough estimation of this left research space's size is 5.625417×10^{39} cycles. With the simple algorithm 4, I did not expect to solve this problem but to get a hamiltonian cycle less than initial cost of the 221435.5555. The reduction of the cost is $164,834.3817$. Because, algorithm 2 is done by hand, the total time of execution can not be estimated. Figure 12 depicts the computer equipment used.

Figures 13, and 17 depict solutions where all moves are knight's path but one is not. The b) edge's cost point out where there is not a knight's move.

Proposition 6.1. *A KTP exists in any chessboard of $m \times n$ squares if only if nm is even.*

Proof. A knight jumps from a white square to a black square. Therefore, in order to complete a hamiltonian cycle with only knight's path, it is necessary to have equal number of white squares and black squares. □

Remark 6.2. Proposition 2.2 states a very strong property but the verification is exhaustive. The previos proposition is easy to verify and together with prop. 2.2 it is immediately that for KTP in $n \times n$ chessboard, n must be greater or equal 4, and even to guaranty the solution of KTP. A version of the previous proposition with black and white domino tiles, also requires nm even in order to be tiling a $n \times m$ chessboard.

Therefore, for 5×5 and 7×7 chessboards there are not hamiltonian cycles with only knight's path. Algorithm 5 is computable because it was designed for a chessboard $m \times n$ squares with nm, even. For clarity, prop. 2.2 states that hamiltonian trajectories exist in any knight graph. The prop. 2.2 and 6.1 are the kind of details that frequently happen in the design of efficient methods for instances of NP like problems. Previous knowledge, formal propositions, or heuristic ideas work very well for a instance problem or case, but, they do not work for similar and related problem. In order to solve, 5×5 and 7×7 chessboards repeat-end control structure for 200 times replaces the while-end in lines 2 through 5. The sorted cost matrix ⇕ in figures 14, and 18 b) depict that there are many similar solutions because not all red dots are on the left side. The value of the final cost in fig. 13, and 17 b) depict a hamiltonian trajectory omitting one edge. The cost values or the edge's cost graphic are indicators to allow us the corroboration of the solution without additional computational cost.

Figure 23 depicts the initial and final tour, and figure 24 depicts that the final tour corresponds only to knight's paths, i.e., $(i - i')^2 + (j - j')^2 = 5$ where (i, j) and (i', j') are the integer coordinates of the squares of the final hamiltonian cycle. For the figure 25 an heuristical estimation of the posibles alternatives or a rough estimation of the research space's size is $29,030,400$.
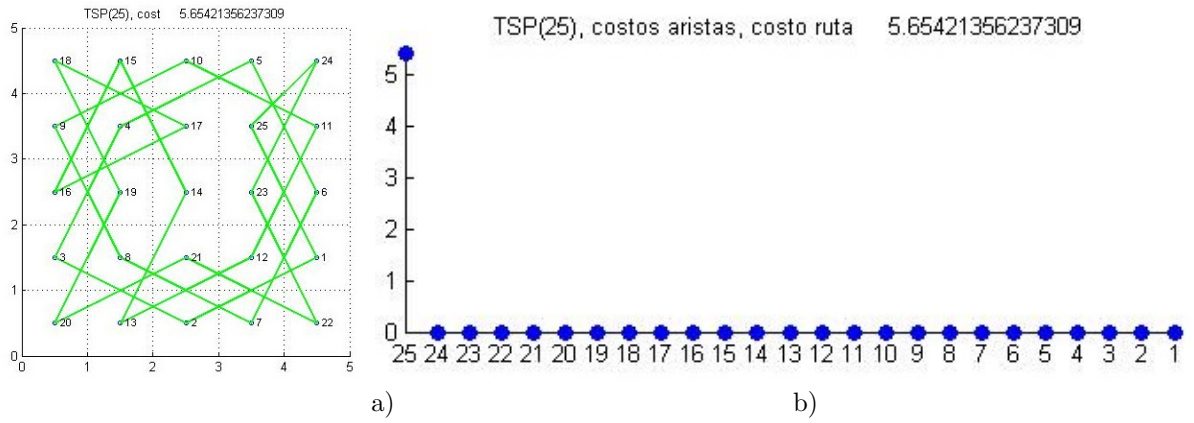
14

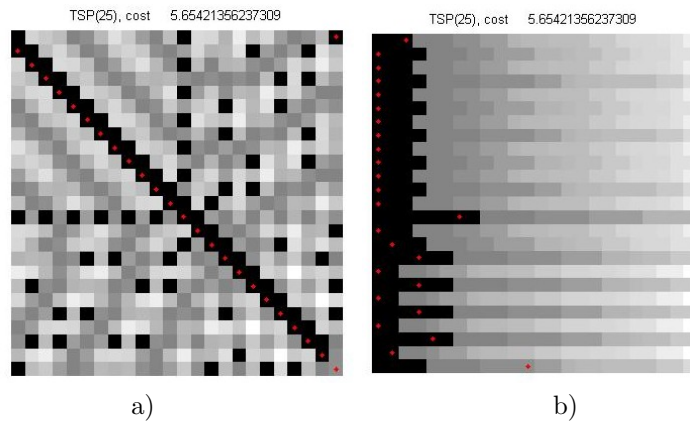Figure 13: TKP$_{5\times5}$. a) final tour, and b) edges'cost.

Figure 14: TKP$_{5\times5}$ of the final tour. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.

Figure 15: TKP$_{6\times6}$. a) final tour, and b) edges'cost.

Figure 16: TKP$_{6\times6}$ of the final tour. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.

Figure 17: TKP$_{7\times7}$. a) final tour, and b) edges'cost.

Figure 18: TKP$_{7\times7}$ of the final tour. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.

Figure 19: TKP$_{8\times8}$. a) final tour, and b) edges'cost.

Figure 20: TKP$_{8\times8}$ of the final tour. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.
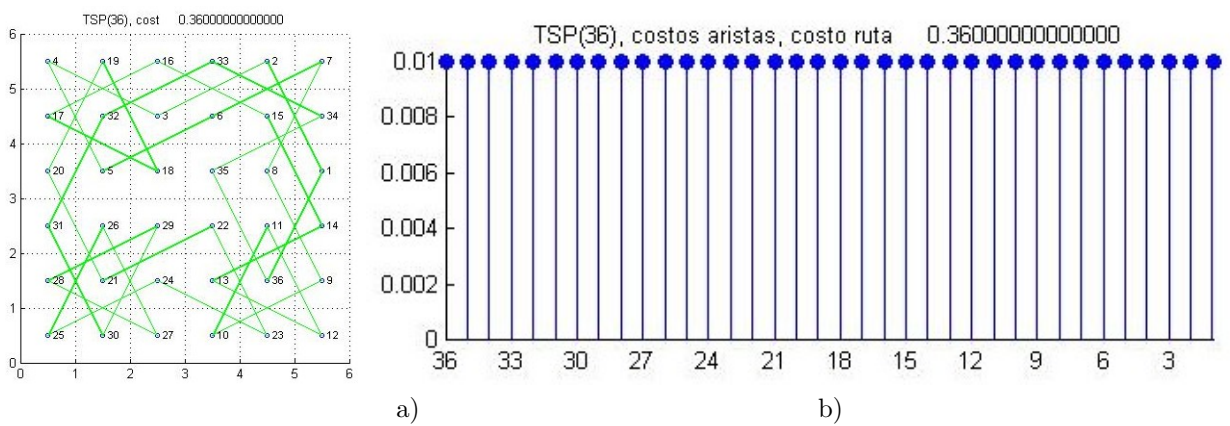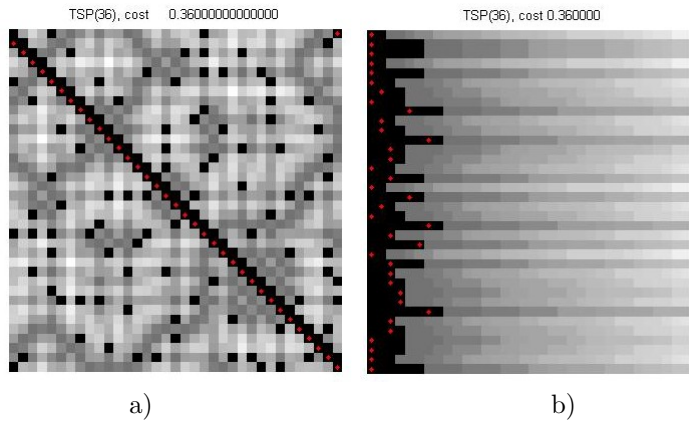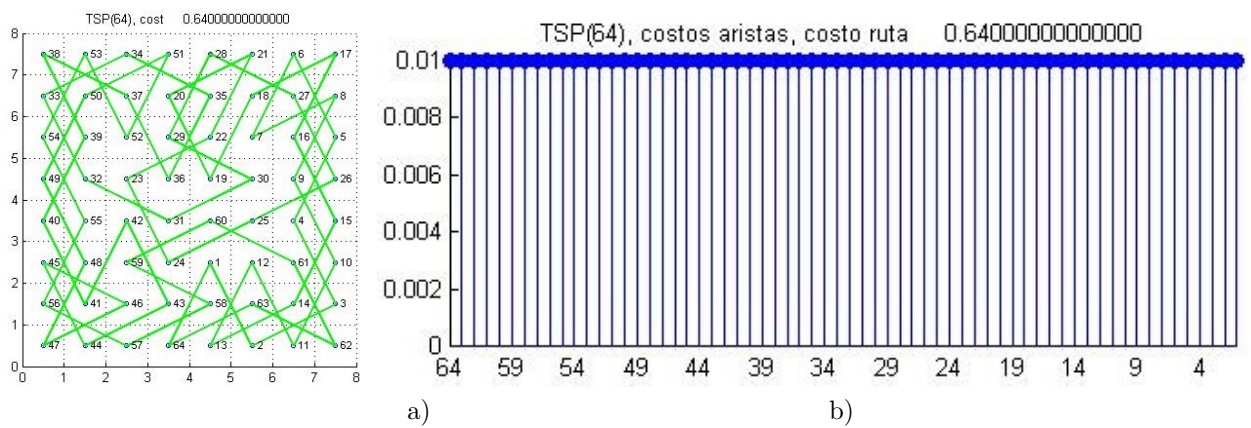

Figure 21: TKP$_{8\times8}$. a) final tour 2, and b) edges'cost.
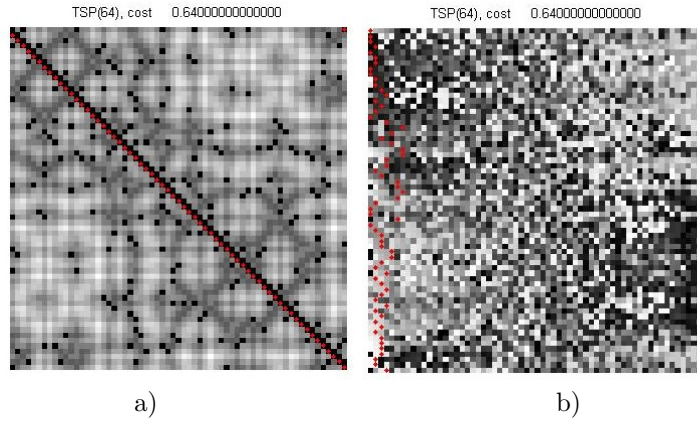
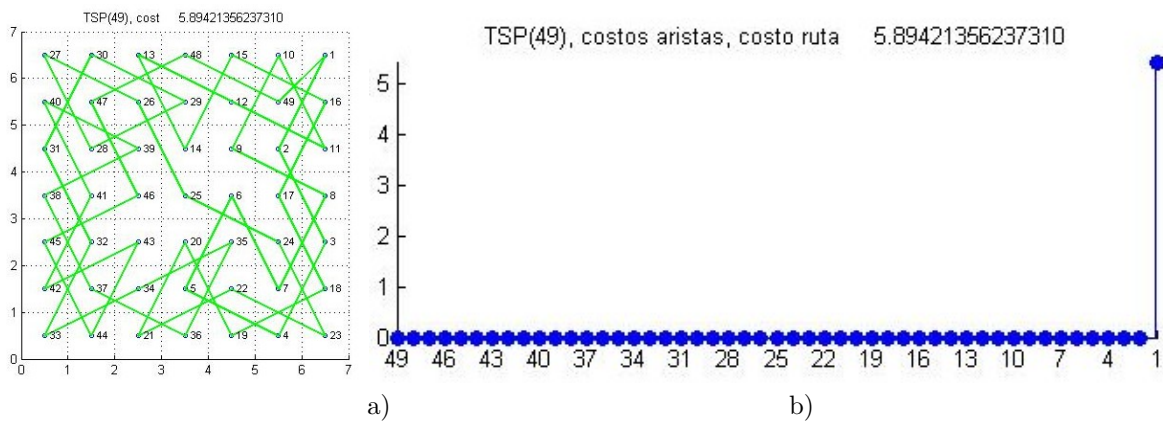Figure 22: TKP$_{8\times8}$ of the final tour 2. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.
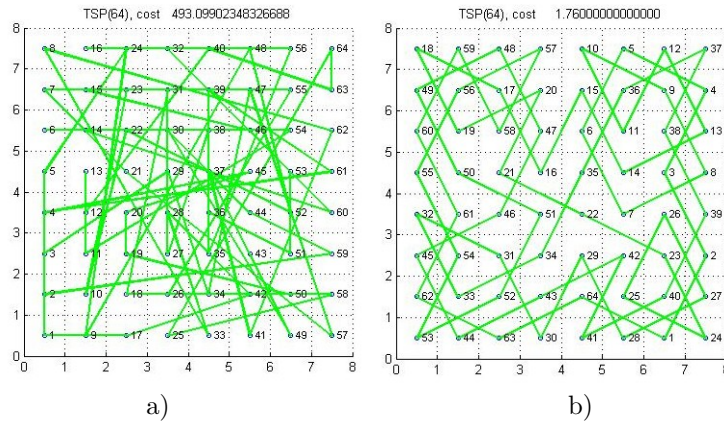
Figure 23: TKP$_{8\times8}$. a) Initial tour, and b) final tour with cost < 4.
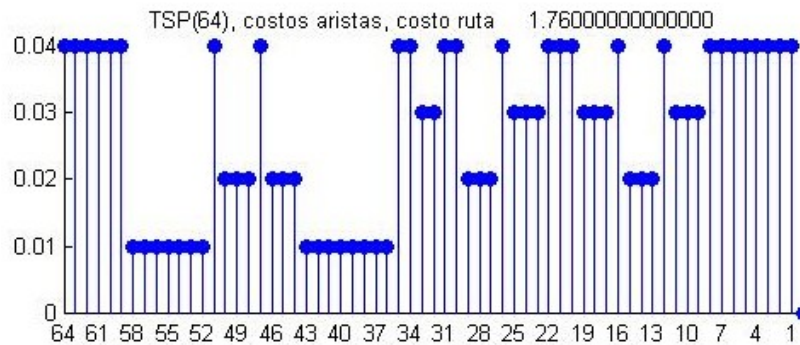
Figure 24: TKP$_{8\times8}$. Edge's cost depicts that they correspond only to knight's motions for the final tour.

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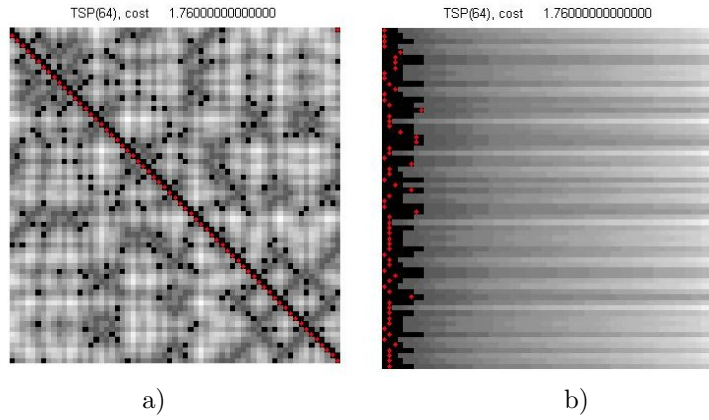
Figure 25: TKP$_{8\times8}$ of the final tour. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.

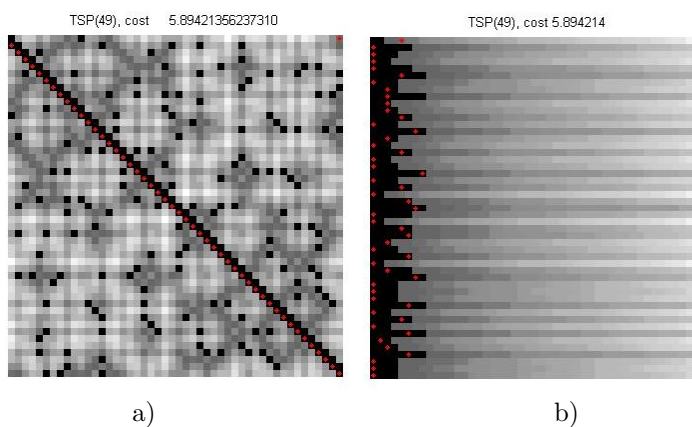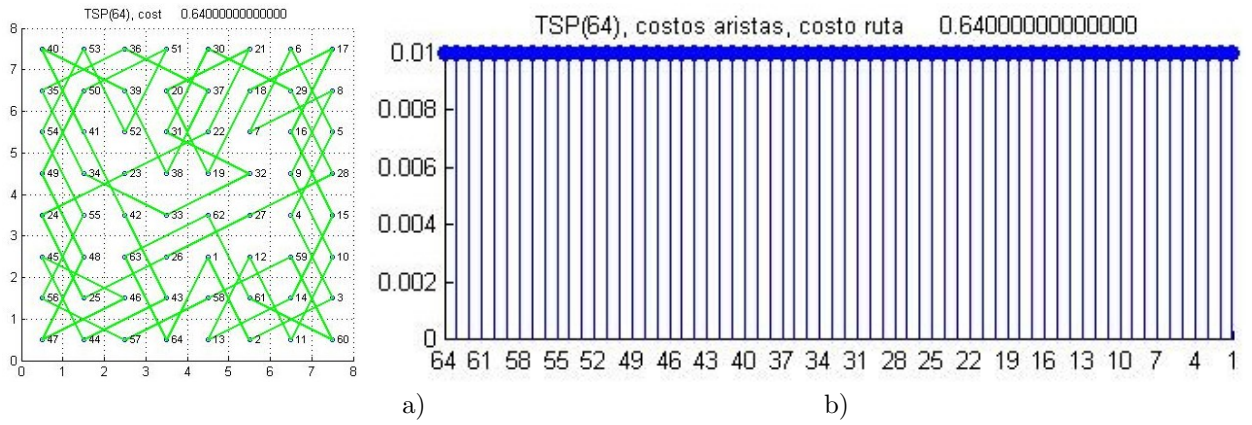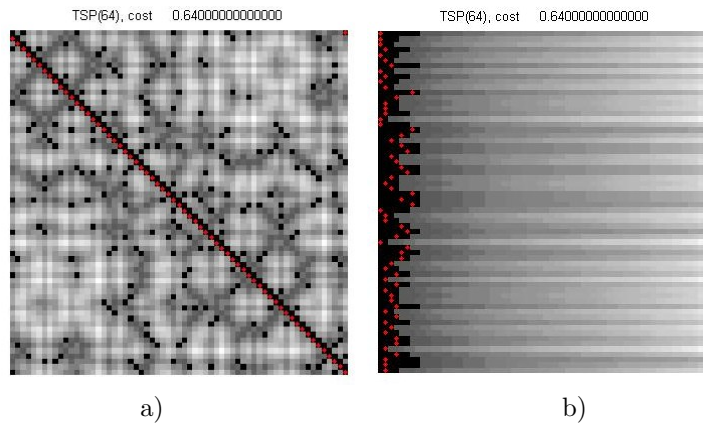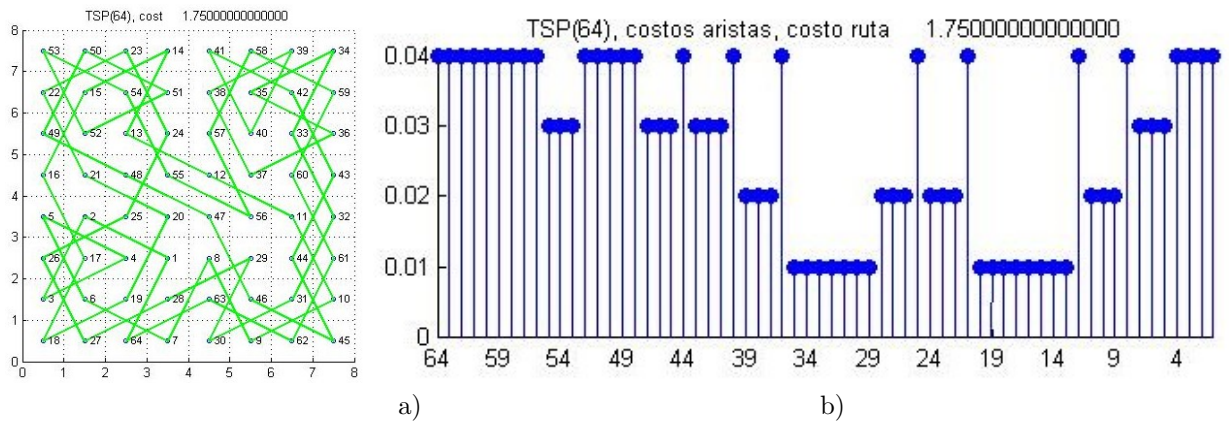Figure 26: TKP$_{8\times8}$. a) final tour 2, and b) edges'cost.
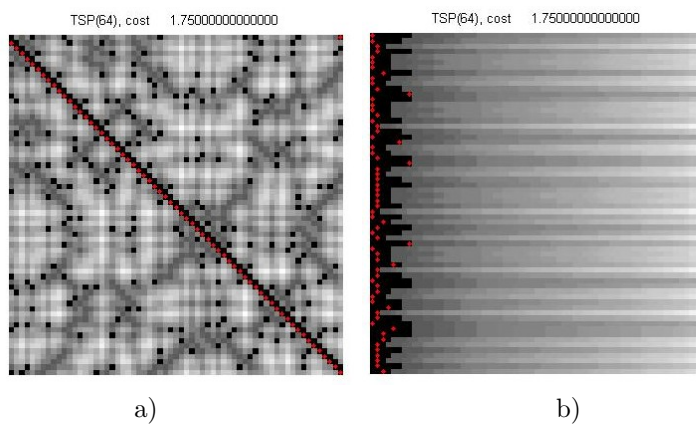
Figure 27: TKP$_{8\times8}$ of the final tour 2. a) Ordered cost matrix, and b) sorted cost matrix $\mathcal{M}$.

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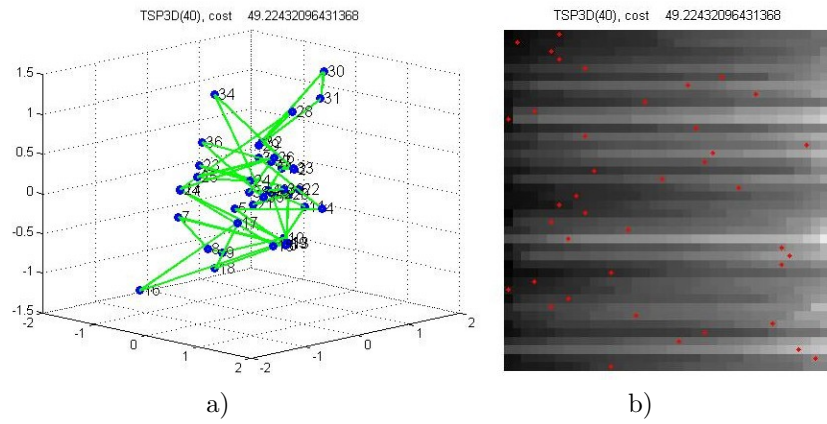
Figure 28: TSP3D$_{40}$. a) Initial 3D Hamiltonian cycle, and b) sorted cost matrix $\mathcal{M}$. The red dots on the left side implies that there are many alternatives hamiltonian cycles.
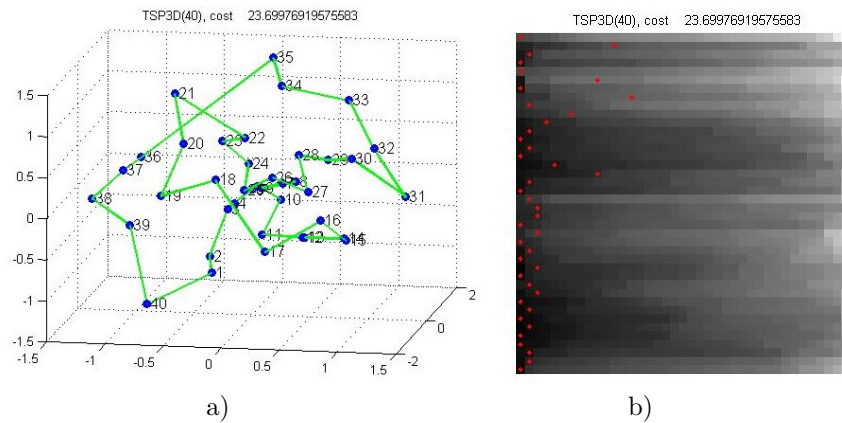
Figure 29: TSP3D$_{40}$. a) Putative 3D Optimal hamiltonian cycle, and b) sorted cost matrix $\mathcal{M}$. The red dots on the left side implies that there are still many alternatives hamiltonian cycles.
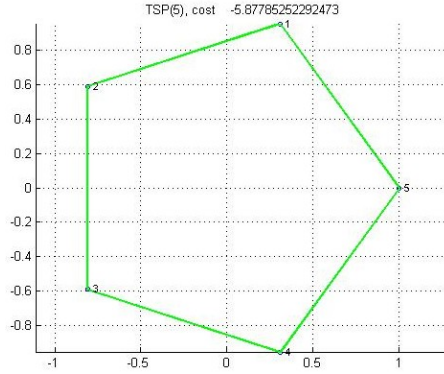
Figure 30: Pentagon as a TSP_5.

7 Polygonal shapes and properties

This section depicts efficient algorithms using previous knowledge to replace the algorithm of section 3. It is clear that the population of the searching space for KTP or TSP problems are reduced when an algorithm focuses in appropriate properties. In fact, the next proposition follows trivially from the prop. 4.4.

Proposition 7.1. *Given a TSP_n where the cities are the vertex of a regular polygon in $\mathbb{R}^2$. Then the minimum Hamiltonian cycle is the polygon using Euclidian,* max*, and abs distance.*

A regular polygon has sides of the same length. Hereafter, regular polygon is named polygon.

An algorithm to state this obvious solution must be $\mathbf{O}(n)$. Figure 30 depicts an example of the polygon as TSP_5. This is by using the formulas $y_i = r\sin(2\pi i/n)$ and $x_i = r\cos(2\pi i/n)$, $i = 0, \ldots, n-1$. The cost of minimum Hamiltonian cycle depends of the Euclidian, max, or abs distance, but not the shape, which is always a Jordan's simple convex curve.

$Polygon_n$ shape problem have a property to work for searching the minimum length, but what about the maximum length on cities as the vertex of a polygon of n vertices in $\mathbb{R}^2$ using Euclidian, max, and abs distance.

Hereafter, $PoMX_n$ define the problem find the maximum distance of the vertices of a polygon of n vertices using Euclidian ($POME_n$), max ($POMm_n$, or abs (($POMa_n$) distance where X denotes E,m or a. Also, an star is a crossing Hamiltonian cycle over the vertices of a polygon with all angles equals.

Proposition 7.2. *Given a $PoME_n$ where the cities are the vertex of a polygon in $\mathbb{R}^2$. Then the maximum Hamiltonian cycle is a star when n is odd.*

*Proof.* First, assuming that the Hamiltonian cycle is a star, it has the maximum length under Euclidian distance because the secants corresponds to the diagonals of circumscribe triangles.

For n odd, without loss of generality $n = 2k + 1$ for some $k \in \mathbb{N}$. The sequence $\{(1 + (i - 1)\,k)\,\mathrm{mod}\,n\}$, $i = 1, 2, \ldots, n, n+1$, form an star (crossing Hamiltonian cycle) for the vertices of the given polygon.

The function $s(i, k) = (1 + (i - 1)\,k, (1 + (i - 1)\,k)\,\mathrm{mod}\,(2k + 1))$ computes a positive integer where the second entry is the residuos of n or the congruent classes mod n. For a given k, two consecutive numbers $1 + (i - 1)\,k$ and $1 + (i)\,k$ are in different class mod n. For $i = n + 1 = 2k + 2$, corresponds to the class 1 mod n. And by construction all sequence numbers $1, 1 + k, \ldots, 1 + (i - 1)\,k$, $i = 1, 2, \ldots, n, n+1$, are in different class mod n except the first and the last one. □

For n even under the Euclidian distance there is not a proposition as the previous but similar to a star when $n >> 0$. In fact, for any n even never is a star. Figure 33 depicts a no star.

For n odd the symmetry allow to build an efficient algorithm under the Euclidian distance. Assuming that the vertices are numbered using the previous formulas for (x_i, y_i), then $1, \left(1 + (i - 1)\,\frac{n-1}{2}\right)\,\mathrm{mod}\,(n), \ldots, 1$ defines a star. By example for $n = 13$ the vertices of the star are $1, 7, 13(\text{or } 0), 6, 12, 5, 11, 4, 10, 3, 9, 2, 8$, and 1 as is depicted in Fig. 31. Figure 34 depicts a star for $POME_{99}$.

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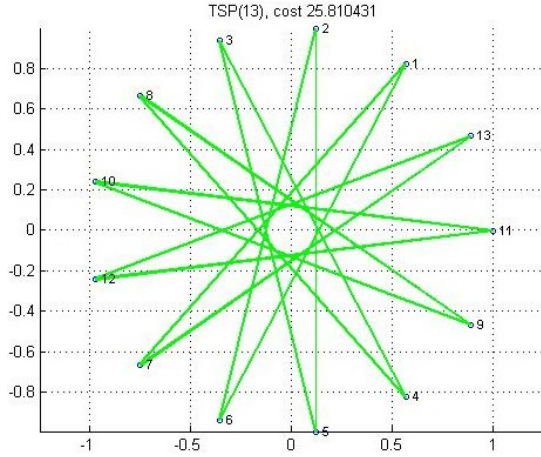
Figure 31: POME$_{13}$ is a star.

a)   b)
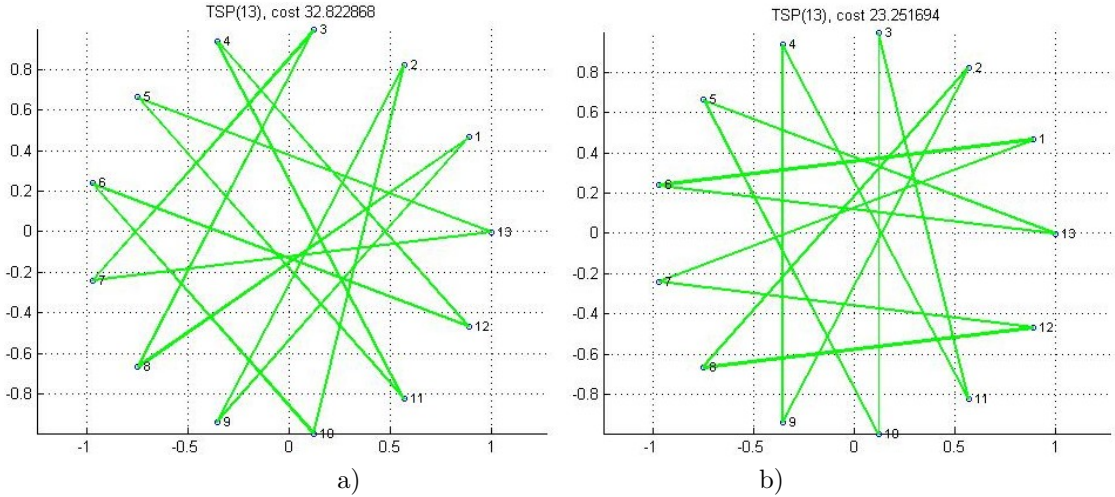
Figure 32: a) POMa$_{13}$ is no star, and b) POMm$_{13}$ is no star.

On the other hand, POMa$_{13}$ and POMm$_{13}$ are no stars, they are depicted in Fig. 32 a) and b) respectively. These differences are caused by using max distance and abs distance instead Euclidian distance.

It is well know that an algorithm for minimization can be used for maximization using $-f$. However, geometry properties, as by example Simple Jordan's curve is not preserve in maximization. Moreover, minimum length over a polygon is always a polygon. This previous knowledge defines a very efficient algorithm to build the minimum Hamiltonian cycle likes an enumeration of the vertices of a polygon. Similar situation is with n odd is for PoME$_n$, where the cities are the vertex of a polygon in $\mathbb{R}^2$. The efficient algorithm corresponds to an enumeration using the index formula $\left(1 + (i-1)\frac{n-1}{2}\right) \bmod (n)$, for $i = 1, \ldots, n+1$. Complexity is $\mathbf{O}(n)$.

As in previous sections, the study of a problem allow to discover properties for building efficient algorithm. The convexity and an appropriate lattices for Lennard-Jones problems (LJP$_n$) are important properties used in [9, 10]. However, these properties can not be generalized or inherited to other NP problems.

8 There is not a property for solving GAP
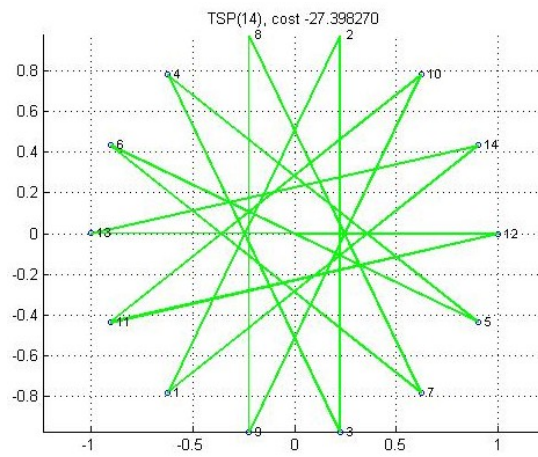
Figure 33: $POME_{14}$ is no star.
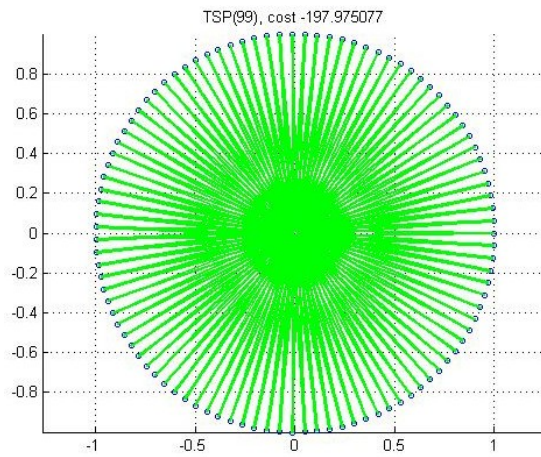
Figure 34: $POME_{99}$ is a star.

The algorithm 1 is blind to the properties of TSP and KTP. Jordan's simple curve and hamiltonian cycles with many cross as posible are clearly oppositive properties.

For KTP, the objective function is adapted for solving a decision problem: is there a hamiltonian cycle for a knight in a chessboard 8×8?

The rippling effects proves that GAP's the objective function giving by the summation os the edges' cost is not monotonic or convex. This can see assuming few $c_{ij} > 0$ and their corresponding $c_{ij} < 0$.

Proposition 8.1. *Given a GAP_n there is $\mathbb{R}^K$ where the vertices of GAP can be located as points in $\mathbb{R}^K$ with the max distance d_m.*

Proof. The unknowns are the coordinates of each vertex. Linear equations can be generated from the edge's cost.

Without loss of generality, I assume that the cost matrix is asymmetrical. For $K > 0, L_j \leq K$, the linear system to solve is:

$$
\begin{aligned}
x_1^1 - x_1^2 + x_2^1 - x_2^2 + \cdots + x_{K-1}^1 - x_{K-L_1}^2 + x_{K-L_1+1}^1 + \cdots + x_K^1 &= c_{1,2} \\
x_1^1 - x_1^2 + x_2^1 - x_2^2 + \cdots + x_{K-1}^1 - x_{K-L_2}^2 - x_{K-L_2+1}^2 + \cdots - x_K^2 &= c_{2,1} \\
\vdots &= \vdots, \\
x_1^n - x_1^{n-1} + x_2^n - x_2^{n-1} + \cdots + x_{K-L_{n^2-1}}^n - x_{K-L_{n^2-1}}^{n-1} + x_{K-Ln^2-1+1}^{n-1} + \cdots + x_K^{n-1} &= c_{n-1,n}, \\
x_1^n - x_1^{n-1} + x_2^n - x_2^{n-1} + \cdots + x_{K-L_{n^2}}^n - x_{K-L_{n^2}}^{n-1} - x_{K-Ln^2+1}^n - \cdots - x_K^n &= c_{n,n-1},
\end{aligned}
$$

where $x^i = (x_1^i, x_2^i, \ldots, x_K^i) \in \mathbb{R}^k$, $i = 1, \ldots, n$ are the corresponding coordinates of the vertices. With sufficient variables, i.e, appropriate values of K and L_j, $j = 1, 2, \ldots, n^2$, the previous linear system is always soluble. $\square$

Remark 8.2. Any GAP can be mapped in $\mathbb{R}^K$ under d_m.

In the core of the algorithms for TSP and KTP is the algorithm 1. It is designed for exploring edges'cost for looking a minimum or for doing a random uniform search. The heuristic used in the algorithm for TSP is well know, but it only works for a plane where the distance fulfil the cosine law of the triangles, i.e., where Prop. 6.11 in [10] holds. Even, in 3D TSP does not comply. Figure 28 depicts an initial hamiltonian cycle for a TSP with 40 cities in a 3D Space with cost = 29.2243. Figure 29 depicts a putative optimal hamiltonian cycle with cost = 23.6998. There is a reduction on the cost, but, both figures b) imply that there are many alternatives to explore yet.

On the other hand, KTP could be solved requiring hamiltonian cycles with length equals to $n \times m \times 5$ with $dist(x, y) = (i - i')^2 + (j - j')^2$, where $x = (i, j)$, and $y = (i', j')$, $1 \leq i, i' \leq n$, and $1 \leq j, j' \leq m$. In this case the objective function could used without changes, but algorithm 1 must be modified to random uniform search (see 3.1).

9 $\text{SAT}_{n \times m}$ has not properties for an efficient algorithm

This section is devoted to a former NP problem. I apply my technique: 1) general problem, 2) simple reduction, and 3) there is no property for building an efficient algorithm for the simple reduction problem.

Here, I use the convention to represent $\overline{x_i}$ to 0 (false), x_i to 1 (true), and 2 when the variable x_i is no present. Let $\Sigma = \{0, 1\}$, $x_i : \Sigma \to \Sigma, x_i(v) = v, v \in \Sigma, \overline{x_i} : \Sigma \to \Sigma, \overline{x_i}(v) =]v, v \in \Sigma$ (logical not).

A particular Boolean Satisfiability Problem (SAT) consists in finding a set of values in Σ of n bolean variables and m formulas to have the following system equal to 1:

$$
\begin{aligned}
&(x_1 \vee \overline{x_2} \vee \cdots \vee x_l \cdots \vee x_n) \\
\wedge &(x_1 \vee x_2 \vee \cdots \vee x_{n-1}) \\
&\vdots \\
\wedge &(\overline{x_2} \vee x_3 \vee \cdots \vee \overline{x_i} \cdots \vee x_l).
\end{aligned}
$$

Then the general problem or complex SAT is where the formulas could have any subset of boolean variables, where each formula can be mapped to a ternary number.

For example:

$$(x_5 \vee \overline{x_4} \vee x_1)$$
$$\wedge (x_3 \vee x_2)$$
$$\wedge (\overline{x_4} \vee x_3 \vee \overline{x_2} \vee x_1).$$

It is traduced to:

$$10221$$
$$22112$$
$$20101.$$

The assignment $x_1 = 1$, $x_2 = 1$ satisfies the previous example. It is not unique. Moreover, the number 22211 is a representation of this assignment as a ternary number. Also, the system:

$$(x_5 \vee \overline{x_4} \vee x_1)$$
$$\wedge (x_3 \vee x_2)$$
$$\wedge (\overline{x_4} \vee x_3 \vee \overline{x_2} \vee x_1)$$
$$\wedge (x_2 \vee x_1).$$

22211 is translated like the last formula and it is a type of a fixed point system, where 22211 is an appropriate assignment. The details are given below.

On the other hand, the simple reduction provides a simple version of SAT, where for convenience all formulas have the same variables. In any case, this simple versión of SAT is like study parts of the complex SAT, and it is sufficient to prove that there is not polynomial time algorithm for the simple SAT, and also, for general SAT.

An equivalent functional formulation is:

Let $\overrightarrow{x} = (x_1, x_2, \ldots, x_n) \in \Sigma^n$, $\overrightarrow{y} \in \Sigma^m$, $S_j \subset \{1, \ldots, n, -1, \ldots, -n\}$, $j = 1, \ldots, m$, $F_j : \Sigma^m \to \Sigma$, $F_j(\overrightarrow{x}) = \vee_{k \in S_j} z_k$, $z_k = \begin{cases} x_k & k > 0 \\ \overline{x}_k & k < 0 \end{cases}$, $G : \Sigma^m \to \Sigma$, $G(\overrightarrow{y}) = \wedge_{i=1}^{m} y_i$

The SAT is

$$G \circ (F_1, \ldots, F_m)(\overrightarrow{x}).$$

Let be $U = [u_{jk}]$ a matrix of $\Sigma^{m \times 2n}$,

$$u_{j,k} = \begin{cases} 1 & \text{if } x_i \vee \overline{x}_i \text{ are variables of } F_j, k = i, \text{if } i > 0, k = n + |i|, \text{if } i < 0, \\ 0 & \text{otherwise.} \end{cases}$$

The matrix U let to know how many variables has F_j as the of summation over the row, $\sum_{i=1}^{2n}(u_{ji})$. Also the summation over a column, $\sum_{j=1}^{m}(u_{jk})$ is the number of times that the x_i or $\overline{x}_i$ ($k = i$, if $i > 0$, or $k = n + |i|$, if $i < 0$) is used in all F_j.

Let be $\blacksquare$ as 0, and $\square$ as 1. The following boards have not a set of values in Σ to satisface them (I called unsatisfactory boards):

	x_2	x_1
x_1	$\blacksquare$	$\blacksquare$
$\blacksquare$	$\blacksquare$	$\square$
$\square$	$\square$	$\blacksquare$
	$\square$	$\square$

Hereafter, a SAT of n bolean variables and m formulas is denoted by SAT$n \times m$.

Proposition 9.1. *Given a $SAT_{n \times 2n}$ where formulas as the squares correspond to the 0 to $2^l - 1$ binary values, is an unsatisfactory board.*

Proof. The binary values from 0 to $2^l - 1$ are all posibles assignation of values for the board. It means that for any possible assignation, there is the oppositive formula with value 0. $\square$

Proposition 9.2. *Given a SAT of $n \times m$, there is not a satisfactory set of values in Σ, when there is a subset of l bolean variables and their set of Fj formulas are isomorphic to an unsatisfactory board.*
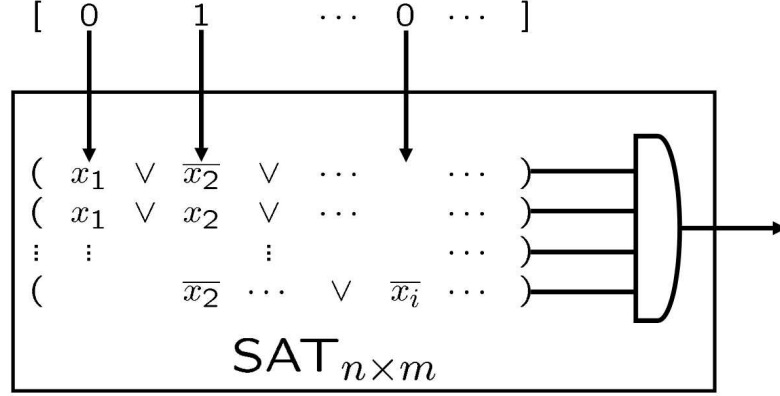
Figure 35: SAT$_{n\times m}$ is a white of box containing a circuit of logical gates.

Proof. It is immediately, the subset of l bolean variables can not satisface their 2^l, F_j formulas. Therefore, there is not posible to find satisfactory set of n values for this SAT. $\qquad\square$

When I focusses in the classical techniques for fixed point or finding roots. The formulation of one can be used in the other. This means $f : \mathbb{R} \to \mathbb{R}$ with a root $x_1^* \in (a, b)$ then the function $g : \mathbb{R} \to \mathbb{R}$, giving by $g(x) = x + f(x)$ has the fixed point x_1^*. The reciprocal, let the function $g : \mathbb{R} \to \mathbb{R}$ has a fixed point x^* then the function $f : \mathbb{R} \to \mathbb{R}$, giving by $f(x) = x - g(x)$ has the point x_1^* as a root.

SAT$_{n\times m}$ can be transformed in a fixed point formulation. This formulation is easy to understand, it consists in:

1. $\Sigma = \{0, 1\}$.

2. giving a boolean variable x is mapped to 1, and $\bar{x}$ to 0;

3. a formula: $\overline{x_{n-1}} \vee \cdots x_l \vee \cdots \vee \overline{x_1} \vee x_0$ can be transformed in its corresponding binary number: $0 \cdots 1 \cdots 01$;

4. if a y is a binary number, $\overline{y}$ is the complement binary number. this is done bit a bit, 0 is complemented to 1 and 1 is complemented to 0.

5. a system of SAT$_{n\times m}$ correspond the set $M_{n\times m}$ of the m binary numbers of its formulas. $M_{n\times m} = \left\{ \begin{matrix} s_{n-1}^1 s_{n-2}^1 \cdots s_1^1 s_0^1, \\ s_{n-1}^2 s_{n-2}^2 \cdots s_1^2 s_0^2, \\ \cdots, \\ s_{n-1}^m s_{n-2}^m \cdots s_1^m s_0^m \end{matrix} \right\}$. Note that the number $s_{n-}^k s_{n-2}^k \cdots s_1^k s_0^k$ correspond to formula k of the SAT$_{n\times m}$, $k = 1, \ldots, m$.

6. a system of SAT$_{n\times m}$ as a boolean function is SAT$_{n\times m} : \Sigma^n \to \Sigma$. The argument is a binary number of n bits.

Remark 9.3. The complexity of the evaluation of The complexity of the evaluation of SAT$_{n\times m}(y = y_{n-1}y_{n-2} \cdots y_1\, y_0)$ is considered to be **O**(1). It is non trivial to consider system of boolean formulas like a circuit of logical gates as it is depicted in figure 35. Hereafter, SAT$_{n\times m}$ correspond to a logic circuit of "and", "or" gates

Proposition 9.4. *Evaluation of SAT$_{n\times m}$ is equivalent to the evaluation of SAT$_{n\times m}(y = y_{n-1}y_{n-2} \cdots y_1 y_0)$, that consists to verify y_i match a digit $s_i^k \in M_{n\times m}$, $\forall k = 1, \ldots, m$, SAT$_{n\times m}(y = y_{n-1}y_{n-2} \cdots y_1 y_0)$=1, otherwise SAT$_{n\times m}(y = y_{n-1}y_{n-2} \cdots y_1 y_0)$=0.*

Proof. It is immediately, let $x_i = v_i$, $i = n, \ldots, 1$ with $v_i \in \Sigma$ an assignation for $SAT_{n \times m}$. If the set of values v_i satisfies $SAT_{n \times m}$ then it means that at least one bolean variable of each bolean formula is 1, therefore $SATn \times m(v_n v_{n-1} \cdots v_2 v_1)$ is 1. By other hand, if $x_i = v_i$ does not satisfice $SAT_{n \times m}$, then at least one formula the $SAT_{n \times m}$ is 0, let assume it is j. This means that no value v_i match any digit of s_i^j for $i = n, \ldots, 1$. Therefore $SAT_{n \times m}(v_n v_{n-1} \cdots v_2 v_1) = 0$.

Reciprocally, $SAT_{n \times m}(y = y_n y_{n-1} \cdots y_2 y_1) = 1$, means that a digit y_i match a digit $s_i^k \in M$, $\forall k = 1, \ldots, m$. The k bolean formulas are 1. Therefore, $SAT_{n \times m}$ is 1 with the assignation $x_i = y_i$. Otherwise, $SAT_{n \times m}(y = y_n y_{n-1} \cdots y_2 y_1) = 0$ implies that at least one binary number of M does not coincide with the digits of y. This means a bolean formula of $SAT_{n \times m}$ is 0, then $SAT_{n \times m}$ is 0. $\qquad\square$

Proposition 9.5. *An equivalent formulation of $SAT_{n \times m}$ is to look for a binary number x^* from 0 to $2^n - 1$.*

1. $x^* \in M_{n \times m}$ and $\overline{x^*} \notin M_{n \times m}$ then $SAT_{n \times m}(x^*) = 1$.

2. $x^* \in M_{n \times m}$ and $\overline{x^*} \in M_{n \times m}$ then $SAT_{n \times m}(x^*) = 0$. If $m < 2^n - 1$ then $\exists y^*, 0 \leq y^* \leq 2^n - 1$ with $\overline{y^*} \notin M_{n \times m}$ and $SAT_{n \times m}(y^*) = 1$.

Proof. 1. How $x^* \in M_{n \times m}$ and $\overline{x^*} \notin M_{n \times m}$, this means that the corresponding formula of x^* is not blocked and for each bolean formula of $SAT_{n \times m}(x^*)$ at least one bolean variable coincides with one variable of x^*. Therefore $SAT_{n \times m}(x^*) = 1$.

2. How $m < 2^n - 1$, $\exists y^*, 0 \leq y^* \leq 2^n - 1$ with $\overline{y^*} \notin M_{n \times m}$. Adding the corresponding formula of y^* to $SAT_{n \times m}$, a new $SAT_{n \times m+1}$ is obtained. By the previous case, the case is proved. $\qquad\square$

This approach is quite forward to verify and get a solution for any $SAT_{n \times m}$. By example given $SAT_{6 \times 4}$, its correspond set $M_{6 \times 4}$:

	$x_5 = 0$	$x_4 = 0$	$x_3 = 0$	$x_2 = 0$	$x_1 = 0$	$x_0 = 0$
	$\overline{x_5} \vee$	$\overline{x_4} \vee$	$\overline{x_3} \vee$	$\overline{x_2} \vee$	$\overline{x_1} \vee$	$\overline{x_0})$
$\wedge($	$\overline{x_5} \vee$	$\overline{x_4} \vee$	$\overline{x_3} \vee$	$\overline{x_2} \vee$	$\overline{x_1} \vee$	$x_0)$
$\wedge($	$x_5 \vee$	$x_4 \vee$	$x_3 \vee$	$x_2 \vee$	$x_1 \vee$	$\overline{x_0})$
$\wedge($	$\overline{x_5} \vee$	$x_4 \vee$	$x_3 \vee$	$\overline{x_2} \vee$	$x_1 \vee$	$x_0)$

	x_5	x_4	x_3	x_2	x_1	x_0
$[\overline{x_0}] \equiv 1$	0	0	0	0	0	0
$[\overline{x_1}] \equiv 1$	0	0	0	0	0	1
$[\overline{x_0}] \equiv 1$	1	1	1	1	1	0
$[\overline{x_2}] \equiv 1$	0	1	1	0	1	1

The left side depicts that $SAT_{6 \times 4}(y = 000000) = 1$. The right side depicts the set $M_{6 \times 4}$ as an array of binary numbers, where $000000 \in M_{6 \times 4}$. The middle column depicts at least one variable that satisfies the boolean formulas of $SAT_{6 \times 4}$. Also, $y = 000000$ can be interpreted as the satisfied assignment $x_5 = 0$, $x_4 = 0$, $x_3 = 0$, $x_2 = 0$, $x_1 = 0$, and $x_0 = 0$.

In fact, the previous proposition gives an interpretation of the SAT as a type fixed point problem.

Proposition 9.6. *Given a $SAT_{n \times m}$, there is a binary number $y \in M_{n \times m}$ such that $\overline{y} \notin M_{n \times m}$ then y is fixed point for $SAT_{n \times m}$ or $SAT_{n \times m+1}(y)$, where $SAT_{n \times m+1}$ is $SAT_{n \times m}$ with adding the corresponding formula of y to $SAT_{n \times m}$.*

Proof. This result follows from the previous proposition. $\qquad\square$

Using the propositions and properties for $SAT_{n \times m}$, a computable algorithm for solving $SAT_{n \times m}$ is:

Algorithm 6. **Input:** $SAT_{n \times m}$.
Output: x^* such that $SAT_{n \times m}(x^*) = 1$ or $SAT_{n \times m+1}(x^*) = 1$ or $SAT_{n \times m}$ is not satisfied.
Variables in memory: $T[0:2^{n-1}]=0$ of boolean. $mi:=2^n$: Integer, $mx:=-1$: Integer; $ct:=0$: Integer;

1. **for** i:=1 to m

2. Translate formula i of $SAT_{n \times m}$ to a binary number k.

3. **if** $T[k]$ not equal 1 **then**

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4.   **if** $\mathrm{SAT}_{n \times m}(k)$ equal 1 **then**

5.   **output:** k is the solution for $\mathrm{SAT}_{n \times m}$.

6.   **stop**

7.   **end if**

8. $\mathrm{T}[k] := 1$;

9. $ct := ct + 1$;

10. $mi := \mathrm{Min}(k,mi)$;

11. $mx := \mathrm{Max}(k, mx)$;

12.   **end if**

13. **end for**

14. **if** mi $== 0$ **and** mx $== 2^{n-1}$ **and** ct $\geq 2^n$ **then**

15.   **output:** There is not solution for $\mathrm{SAT}_{n \times m}$.

16.   **stop**

17. **end if**

18. **if** $mi > 1$ **and** $\mathrm{SAT}_{n \times m}(mi - 1) == 1$ **then**

19.   **output:** $mi - 1$ is the solution for $\mathrm{SAT}_{n \times m+1}$ with the corresponding formula for $mi - 1$.

20.   **stop**

21. **end if**

22. **if** mx $< 2^n - 1$ **and** $\mathrm{SAT}_{n \times m}(mx + 1) == 1$ **then**

23.   **output:** $mx + 1$ is the solution for $\mathrm{SAT}_{n \times m+1}$ with the corresponding formula for $mx + 1$.

24.   **stop**

25. **end if**

26. **for** i:=0 to $2^n - 1$

27.   **if** $\mathrm{T}[i]$ not equal 1 **then**

28.   **output:** i is the solution for $\mathrm{SAT}_{n \times m+1}$ with the corresponding formula for i.

29.   **stop**

30.   **end if**

31. **end for**

    Assuming that $\mathrm{SAT}_{n \times m}$ is a logic circuit (see remark 9.3) the complexity of its evaluation is $\mathbf{O}(1)$. This is a non trivial assumption that encloses that the boolean formulas of a given $\mathrm{SAT}_{n \times m}$ are given but not analyzed.

    At the end of the previous algorithm, $\mathrm{SAT}_{n \times m}$ is equivalent to set of binary numbers $\mathrm{M}_{n \times m}$. The knowledge for a given $\mathrm{SAT}_{n \times m}$ depends at least from exploring $\mathrm{M}_{n \times m}$. Before of exploring $\mathrm{SAT}_{n \times m}$ no binary numbers associated with it are know. Now, let $\mathbb{K}_{n \times m}$ be knowledge of a given $\mathrm{SAT}_{n \times m}$. $\mathbb{K}_{n \times m} = \{y^*\} \cup \mathrm{M}_{n \times m} \cup \mathbb{S}_{n \times m}$, where y^* is the satisfying number for $\mathrm{SAT}_{n \times m}$, and $\mathbb{S}_{n \times m} = \mathrm{M}_{n \times m}^c$, it is the set of binary numbers that satisfies $\mathrm{SAT}_{n \times m}$. Using the previous proposition, without lost of generality, $y^* \in \mathrm{M}_{n \times m}$.

Proposition 9.7.   *Given a $SAT_{n\times m}$, and $\mathbb{K}_{n\times m}$:*

1. *It is trivial to solve $SAT_{n\times m}$.*

2. *It is trivial to solve $SAT_{n\times m+1}$ from $M_{n\times m} \cup \{\overline{y}^*\}$.*

Proof.

1. y^* solves $\text{SAT}_{n\times m}$.

2. Any $s \in \mathbb{S}_{n\times m} \setminus \{\overline{y}^*\}$ solves $\text{SAT}_{n\times m+1}$.

$\square$

The previous proposition depicts the condition needed to solve efficiently $\text{SAT}_{n\times m}$, which is $\mathbb{K}_{n\times m}$ the knowledge associated with the specific given $\text{SAT}_{n\times m}$.

It is not complicate and complex to create an object data structure combined index array and double links, the drawback is the amount of memory needed to do the insertion and erase with cost $\mathbf{O}(k)$ (k very small integer) by updating only the links.

Assuming that the initial states of the structures for $\mathbb{S}$ and M are giving, it cost for building them can be neglected or considered a constant. Of course the amount of memory needed is exponential (2^n), where n is the numbers of boolean variables for $\text{SAT}_{n\times m}$

The next tables depict that inserting and erasing binary numbers into the structures $\mathbb{S}$ and M only consists on updating links with fixed complexity $\mathbf{O}(k)$ (k very small integer comparing with 2^n).

Est	prev	next
$\mathbb{S}$	8	1
M	0	0

	$\mathbb{S}_{n\times m}$				$M_{n\times m}$		
i	n	prev_i	next_i	i	n	prev_i	next_i
1	000	8	2	1	- - -	0	0
2	001	1	3	2	- - -	0	0
3	010	2	5	3	- - -	0	0
4	011	0	0	4	- - -	0	0
5	100	3	6	5	- - -	0	0
6	101	5	7	6	- - -	0	0
7	110	6	8	7	- - -	0	0
8	111	7	1	8	- - -	0	0

Erasing 011 from $\mathbb{S}_{n\times m}$ and inserting it in $M_{n\times m}$

Est	prev	next
$\mathbb{S}$	8	1
M	4	4

	$\mathbb{S}_{n\times m}$				$M_{n\times m}$		
i	n	prev_i	next_i	i	n	prev_i	next_i
1	000	8	2	1	- - -	0	0
2	001	1	3	2	- - -	0	0
3	010	2	5	3	- - -	0	0
4	- - -	0	0	4	011	4	4
5	100	3	6	5	- - -	0	0
6	101	5	7	6	- - -	0	0
7	110	6	8	7	- - -	0	0
8	111	7	1	8	- - -	0	0

Erasing 001 from $\mathbb{S}_{n\times m}$ and inserting it in $M_{n\times m}$

	$\mathbb{S}_{n \times m}$				$\mathrm{M}_{n \times m}$		
i	n	prev_i	next_i	i	n	prev_i	next_i
1	000	8	3	1	- - -	0	0
2	- - -	0	0	2	001	4	4
3	010	1	5	3	- - -	0	0
4	- - -	0	0	4	011	2	2
5	100	3	6	5	- - -	0	0
6	101	5	7	6	- - -	0	0
7	110	6	8	7	- - -	0	0
8	111	7	1	8	- - -	0	0

Est	prev	next
$\mathbb{S}$	8	1
M	4	2

Erasing 000 from $\mathbb{S}_{n \times m}$ and inserting it in $\mathrm{M}_{n \times m}$

	$\mathbb{S}_{n \times m}$				$\mathrm{M}_{n \times m}$		
i	n	prev_i	next_i	i	n	prev_i	next_i
1	- - -	0	0	1	000	4	2
2	- - -	0	0	2	001	1	4
3	010	8	5	3	- - -	0	0
4	- - -	0	0	4	011	2	1
5	100	3	6	5	- - -	0	0
6	101	5	7	6	- - -	0	0
7	110	6	8	7	- - -	0	0
8	111	7	3	8	- - -	0	0

Est	prev	next
$\mathbb{S}$	8	3
M	4	1

Using the structures, propositions and properties for $\mathrm{SAT}_{n \times m}$, a computable algorithm for solving $\mathrm{SAT}_{n \times m}$ and building $\mathbb{K}_{n \times m}$ is:

Algorithm 7. **Input:** $\mathrm{SAT}_{n \times m}$.

Output: Y set of binary numbers, such that when Y$\neq \{\}$, any $y* \in$ Y, $\mathrm{SAT}_{n \times m}(y^*) = 1$. The structure $\mathbb{S}_{n \times m}$, such that any $x^* \in \mathbb{S}_{n \times m}$, $\mathrm{SAT}_{n \times m+1}(x^*) = 1$. $\mathbb{K}_{n \times m}$ (object structures: M, $\mathbb{S}_{n \times m}$ and Y).

Variables in memory: Y $= \{\}$: set of binary numbers, initialized to empty; Initialized object structures: M, $\mathbb{S}_{n \times m}$.

1. **for** i:=1 to m

2.   Translate formula i of $\mathrm{SAT}_{n \times m}$ to a binary number k.

3.   **if** $\mathbb{S}.\mathbb{S}_{n \times m}[k]$ not equal "- - -" **then**

4.   **if** $\mathrm{SAT}_{n \times m}(k)$ equal 1 **then**

5.   **output:** $y^* = k$ is a solution for $\mathrm{SAT}_{n \times m}$.

6.   **insert**(k, Y).

7.   **delete**($k, \mathbb{S}_{n \times m}$)

8.   **insert**($k, \mathrm{M}_{n \times m}$)

9.   **else**

10.   **delete**($k, \mathbb{S}_{n \times m}$)

11.   **insert**($k, \mathrm{M}_{n \times m}$)

12.   **delete**($\bar{k}, \mathbb{S}_{n \times m}$)

13.   **insert**($\bar{k}, \mathrm{M}_{n \times m}$)

14.   **end if**

15.   **end if**

16. **end for**

At this point, $SAT_{n \times m}$ is equivalent to set of binary numbers $M_{n \times m}$. And $\mathbb{K}_{n \times m}$ has information to solve $SAT_{n \times m}$ immediately.

The knowledge for a given $SAT_{n \times m}$ depends of exploring $M_{n \times m}$. Before of exploring $SAT_{n \times m}$ no binary numbers associated with $\mathbb{K}_{n \times m}$.

Now, let $\mathbb{K}_{n \times m}$ be knowledge of $SAT_{n \times m}$. $\mathbb{K}_{n \times m} = Y \cup M_{n \times m} \cup \mathbb{S}_{n \times m}$, where Y is the set of satisfying number for $SAT_{n \times m}$, and $\mathbb{S}_{n \times m} = (Y \cup M_{n \times m})^c$, it is the set of binary numbers that satisfies $SAT_{n \times m}$.

Remark 8. Having $\mathbb{K}_{n \times m}$, the knowledge of $SAT_{n \times m}$, it is easy to solve any modified $SAT_{n \times m}$. It is need to keep tracking of changes in M and $\mathbb{S}$, then modified SAT can be solved by the proposition 9.7.

So far, $SAT_{n \times m}$ is now a fixed point type problem over a set of binary numbers. It consists in look for a number in $M_{n \times m}$ which is not blocked or to pick up any element $s \in \mathbb{S}_{n \times m}$ when $\mathbb{K}$ is given.

If all number of $M_{n \times m}$ are blocked then the first binary number s from 0 to $2^n - 1$ which is not in $M_{n \times m}$ is the solution for $M_{n \times m}$, i.e. any element $s \in \mathbb{S}_{n \times m}$. This is trivial when knowledge is given or can be created in an efficient way. This results of this section are related to my article [10], where I stated that NP problems need to look for its solution in an search space using a Turing Machine: "It is a TM the appropriate computational model for a simple algorithm to explore at full the GAPns research space or a reduced research space of it". As trivial as it sound, pickup a solution for $SAT_{n \times m}$ depends of exploring its m boolean equations.

The question if there exists an efficient algorithm for any $SAT_{n \times m}$ now can be answered. The algorithm 7 is technologically implausible because the amount of memory needed. However, building $\mathbb{K}_{n \times m}$ provides a very efficient telephone algorithm (see proposition 8, and remarks 5 in [9]) where succeed is guaranteed for any $s \in \mathbb{S}_{n \times m}$. On the other hands, following [9]) there are tree possibilities exhaustive algorithm (exploring all the searching space), scout algorithm (previous knowledge or heuristic facilities the search in the searching space), and wizard algorithm (using necessary and sufficient properties of the problem).

The study of NP problems depicts that only for an special type of problem exists properties such as Jordan's simple curve or n odd for polygon for TSP2D, or convexity, IF lattice, or CB lattice for Lennard Jones structural and potential minimization problems for building an ad-hoc efficient algorithms, but these properties can not be generalizad for any GAP or any member of class NP.

The boolean formulas of $SAT_{n \times m}$ correspond a binary numbers in disorder (assuming an order of the boolean variables as binary digits). The algorithm 6 has a complexity of the size of m (the numbers of formulas of SAT).It is not worth to considered sorting algorithm because complexity increase by a factor $\mathbf{O}(n(2^n))$ when $m \approx 2^n$.

Without exploring, given $SAT_{n \times m}$ then build $M_{n \times m}$. M can be consider an arbitrary set of numbers. The numbers in M have not relation or property to point out the satisfied binary number, in fact one or many numbers could be solution or not one in M, but in the range from 0 to $2^n - 1$ there is a solution or not depending of M. Of course, this is true for an arbitrary set of numbers.

On the other hand, if the formulas of a SAT problem are formulated assuming, by example, no boolean formula is complement of other one, then any translation to binary number is a satisfied assignment.

Hereafter, for the sake of my argumentation the boolean formulas of $SAT_{n \times m}$ are considered a translation to binary numbers in disorder and without any correlation in $M_{n \times m}$.

Proposition 9.8. *With algorithm 6:*

1. *A solution for $SAT_{n \times m}$ is efficient when $m << 2^n$.*

2. *A solution for $SAT_{n \times m}$ is not efficient when $m \approx 2^n$.*

Proof. It follows from algorithm 6. □

In order to find a solution for SAT, there are a probabilistic algorithm rather than algorithm 6. By example a simple probabilistic algorithm for $SAT_{n \times m}$ is:

Algorithm 9. **Input:** $SAT_{n \times m}$.
    Output: x^* such that $SAT_{n \times m}(x^*) = 1$ or $SAT_{n \times m+1}(x^*) = 1$ or $SAT_{n \times m}$ is not satisfied.
    Variables in memory: T[0:2^{n-1}]=0 of boolean. $ct:=0$: Integer;

1. **while (1)**

2.   **Select randomly k in $[0, 2^n - 1]$ minus marked T[i]==1;**

3.   **if $\text{SAT}_{n \times m}(k) == 1$ then**

4.   **output:** k is the solution for $\text{SAT}_{n \times m}$.

5.   **stop**

6.   **end if**

7.   **if $\text{T}[k] <> 1$ then**

8.   T[k]:=1;

9. $ct := ct + 1$;

10.   **if $ct \geq 2^n$ then**

11.   **output:** There is not solution for $\text{SAT}_{n \times m}$.

12.   **stop**

13.   **end if**

14. **end while**

Proposition 9.9. *With algorithm 9:*

1. *A solution for $SAT_{n \times m}$ is efficient when $m << 2^n$.*

2. *A solution for $SAT_{n \times m}$ is not efficient when $m \approx 2^n$.*

Proof.

1. It follows from algorithm 9 for finding k and $m << 2^n$. It implies, for many k that $\text{P}(\text{SAT}_{n \times m}(k)==1) \approx 1$.

2. It follows from algorithm 9 for finding k and $m \approx 2^n$. Here, it is possible that many selected randomly number in $[0, 2^n - 1]$ are blocked. Therefore $\text{P}(\text{SAT}_{n \times m}(k)==0) \approx 1$. This could cause that algorithm 9 does not solve $\text{SAT}_{n \times m}$ in an appropriate amount of time.

$\square$

Finally,

Proposition 9.10. *$SAT_{n \times m}$ has not property or heuristic to build an efficient algorithm.*

Proof. Given $\text{SAT}_{n \times m}$, its translation is a set of binary numbers, in disorder and without any any previous knowledge, nor correlation in $\text{M}_{n \times m}$. If such property or heuristic exist then any arbitrary subset of number has it. Such characteristic or property must imply that any natural number is related to each other. This means it is in the intersection of all properties for all natural numbers. Also it is no related to the value, because by example, the intersection of natural clases under the modulo of a prime number is empty. Moreover, it is something a priory, otherwise, it will imply to revise all number at least in $\text{M}_{n \times m}$. Using it in a random algorithm, to point out efficiently to the number, which is solution, means that this number is inherently not equally probable. Also, this binary number correspond to an arbitrary arrangement of the boolean variables of $\text{SAT}_{n \times m}$, so in a different arrangement of the positions for the boolean variables, any number is inherently not equally probable! $\square$

Proposition 9.11. *NP has not property or heuristic to build an efficient algorithm.*

Proof. Let be X a problem in NP. PH_X is the set of properties or heuristics for building an efficient algorithm for problem X.

$$\bigcap_X PH_X = \{\}$$

by the previous proposition. □

Remark 10. My previous work was over GAP and NP Hard. In short , let be Y a NP Hard problem, and PHY is the set of properties or heuristics to solve Y. I state:

$$\bigcap_Y PH_Y = \{\}.$$

This does not imply NP is not P but proposition 9.11 does.

Conclusions and future work

In [10], I concluded: "It could be possible to find a reduced research space using an appropriate triangulation so that the global solution does not escape. But Triangulation algorithms has $\mathbf{O}\left(n^{\frac{m-1}{p}}\right)$, $\forall p \in [2, m-1]$ (see [3]). Therefore, it is highly possible that it exists an algorithm to solve arbitrary and large mD Euclidian TSP_n with large polynomial time until some finite $m > 0$."

I was impressed for computing in a very fast way a general triangulation in $\mathbb{R}^m$, that could allow to define an efficient algorithm. Also, I point out that there is not an efficient verification algorithm for he solution of a NP Hard problem (GAP is a minimization problem) instead of focusing in decision NP problem as SAT.

On the other hand, Jordan's simple curve is an example of a property to create an efficient algorithm for a necessary type of solution of a NP hard problem. It is for solving approximately Euclidian Travelling Salesman problem in 2D planes but not in $\mathbb{R}^m$ spaces, $m > 2$. Here the Euclidian metric implies that the quadrilateral's sides are less than the quadrilateral's diagonals. Polygon or convex distribution of the cities, Jordan is a sufficient and necessary property, this previous knowledge allows to build efficient algorithms. For $\mathbb{R}^m$, even with fast triangulations, this means more alternatives to explore, without a property to reduce global complexity in the corresponding searching space.

Heuristic techniques, using previous knowledge of a problem in $\mathbb{R}^m$ do not provide reducibility (see 6 in [10]) for any NP problems.

There are nice properties Jordan's simple curve and star. Any $POME_n$, $n > 3$ odd has a solution type star. Any TSP2D has a solution as a Jordan's simple curve. Crossing lines in a solution is the opposite of not crossing lines in other type of problem.

For LJ problems or for the Searching of the Optimal Geometrical Structures of clusters of n particles remains out of the existence of an efficient algorithm, the selection of the n particles is the bottle's neck (see [9]).

Here, the classical SAT (a NP decision problem) provides a general case, where formulas have any number of boolean variables, then it is is reduced to a simple version $SATn \times m$, all boolean formulas have the same number of boolean variables. This allows to see that does not exist a property for solving SAT with reducibility of the searching space. This shows that there is not a general property that allows to solve SAT problems in polynomial time. The classical SAT depicted in section 9 has clearly no properties for solving efficiently it when $m \approx 2^n$.

For the future, I believe that the implementation of the algorithm 2 for TSP brings an efficient time's reduction for finding the optimal hamiltonian cycle under euclidian distance versus looking to solve a global optimization problem without a necessary stop condition.

In [9] are tree type of algorithm:

1. Exhaustive algorithm. It explores all the searching space of a problem. Nothing scape of it, but it is usually very expensive approach.

2. Scout algorithm. It uses previous knowledge or heuristics to solve a problem using some kind of compass. An example, Genetic and Evolutionary programs.

3. Wizard algorithm. It uses necessary or sufficient properties of the problem. Computer programs that uses mathematical properties, by example, optimization programming, linear programming, simplex program, optimization for variational problems, and so on.

Because of the complexity of using Mathematics and Theory of Computation, my perception for solving problem and developing software is that people favor 2) over 3) and 1). My next article for reviewing languages, automates and nowadays computation models argue about it, to turn back to 3) and clever approaches for 1). This is quite important for building reliable and efficient software. Computer Science has a serious crisis by the lack of academic leaders for divulging the computers science and technology. There could be a future crisis human-hardware-software but, "Singularity" (Ray Kurzeil), "The Fourth Paradigm" (Jim Gray) and similar texts need to be carefully and seriously studied for stating a true and objective perspective, to say at least. Herbert Simon (1916-2001, Economy Nobel Winner) predicted in 1957: "Within ten years, a digital computers will discover and prove an important mathematical theorem". Also, he predicted that a computer will be a chess champion by 1967. Gasparov was apparently defeated by Deep Blue at 1997. But, the program of Deep Blue was not human independent, reviewing the article "IBMS DEEP BLUE CHESS GRANDMASTER CHIPS", I found the phrase: "In the 1997 match, we didnt use the hardware escape hatch, but **we did change the weights for the positional features between games**" (I set on bold some a part of this phrase). Fortunately, Herbert Simon and followers are still wrong.

Quantum Computation is on the future road. Even in my university, UAM-Azcapotzalco, I encouraged an student to do his master thesis in this subject: "Classical Realization of Grover's Quantum Search Algorithm using Toffoli Gates". To my knowledge the algorithm 6 can be adapted for using quantum variables for exploring $[0, 2^n - 1]$ states at the same time, instead of the cycle **for** i:=0 to $2^n - 1$. This implies that the complexity for solving NP problems can be reduced to one cycle over a quantum variable. NP problems can be solved efficiently using quantum computation! This open a question that I explore in my next article for reviewing languages, automates and nowadays computation models for depicting that today the human being is an intelligent being but no parallel super computer yet. Human being and quantum computation seem to have the ability for instantaneous exploring.

Finally, this reformulation of my previous works, using Jordan's simple curve versus hamiltonian cycles with many crossings as possible, and the research of the properties of $\mathrm{SAT}_{n \times m}$ supports and proves that there is not a general property to reduce the complexity of the worst case NP problem, i.e., NP $\neq$ P.

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