Complexity of the Mixed Postman Problem with Restrictions on the Arcs

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Mixed Graphs and Postman Tours

- A *mixed graph* $M = (V, E, A)$ consists of sets V of *vertices*, E of *edges*, and A of *arcs*.
- A *postman tour* of M is a closed walk W that traverses all vertices, edges, and arcs of M.
- An *edges postman tour* of M traverses each arc of M exactly once.

An Edge Postman Tour

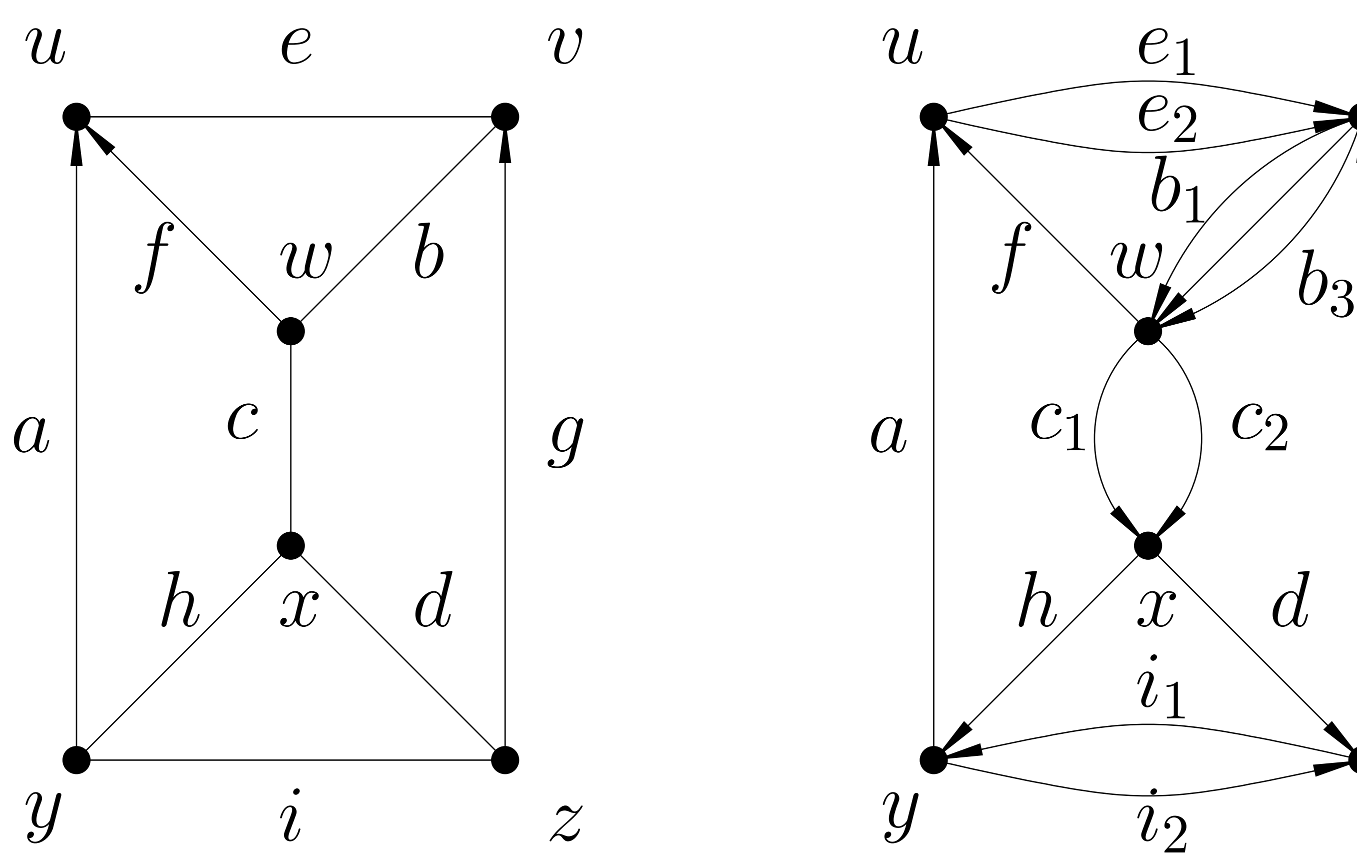

Figure 1: Mixed graph and edges postman tour

Edges Postman Problem

- Given a strongly connected mixed graph $M = (V, E, A)$ and a vector $c \in \mathbb{Q}_+^E$, the *cost* of an *edges postman tour* T of M is the sum of the costs of the edges traversed by T.
- **Problem:** Obtain the minimum cost MEPT(M, c) of an edges postman tour of M.

Reformulation

- We can remove the arcs from the description of the problem.
- For each $v \in V$, let $b_v = d_A(v) - d_A(\bar{v})$ be the *demand* at vertex v.
- An edges postman tour of G is (almost) equivalent to a feasible flow on the directed graph $\vec{G}$ with vector of demands b.
- **Problem:** Obtain the minimum cost MEPT(G, b, c) of an edges postman tour of (G, b).

A Feasible Flow

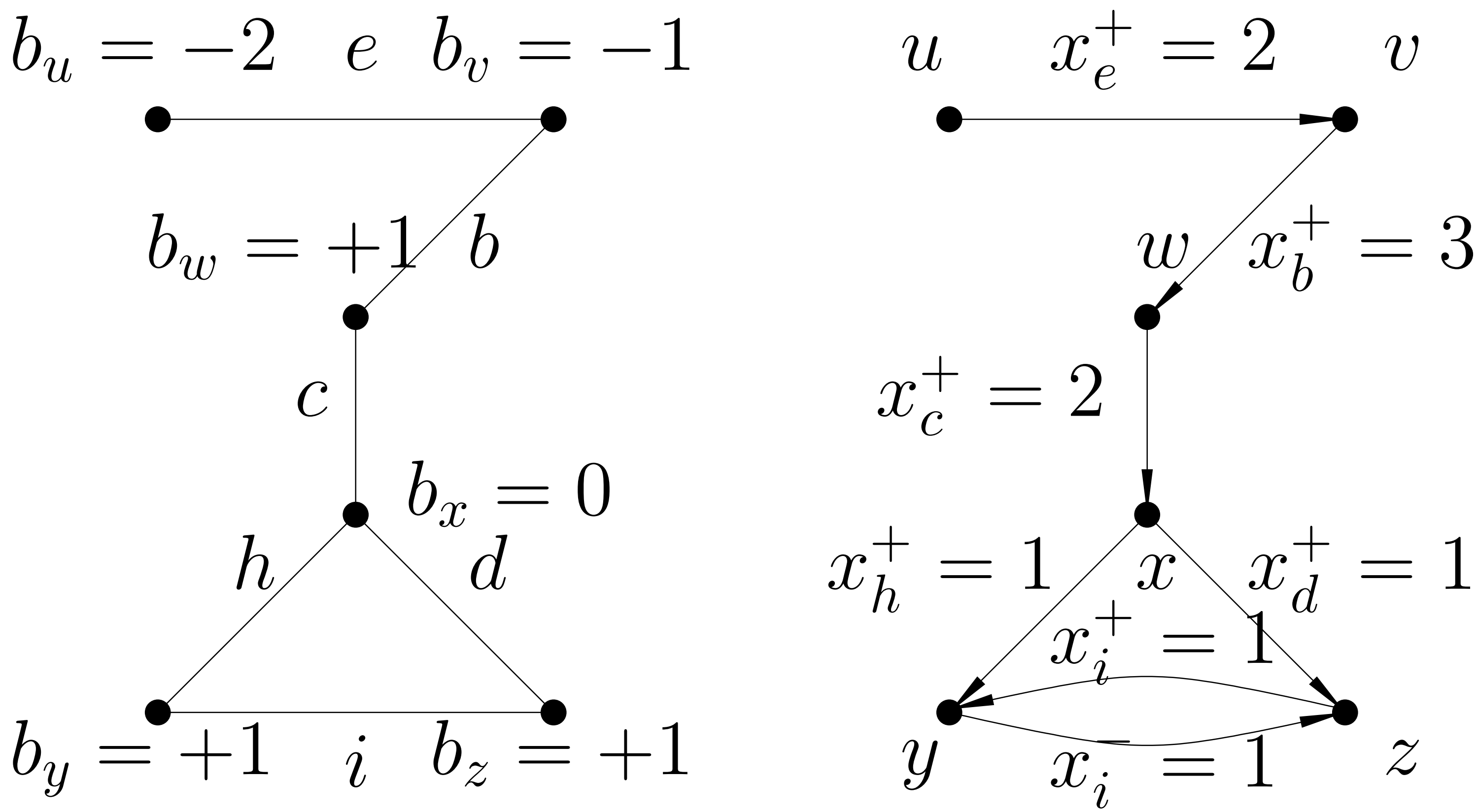

Figure 2: A feasible flow on an undirected graph

Computational Complexity

- The decision version of the edges postman problem is NP-complete (reduction from 1-in-3 satisfiability).
- The edges postman problem can be solved in polynomial time if G is series-parallel or if G is Eulerian.

Integer Programming Formulation

$$\begin{aligned}
\text{MEPT}(G,b,c) &= \min c^\top x \\
\text{subject to} & \\
x(\delta_E(S)) &\geq b(S) \text{ for all } S \subseteq V \\
x(\delta_E(v)) &\equiv b_v \text{ (mod } 2) \text{ for all } v \in V \\
x(\delta_E(S)) &\geq l(\delta_E(S)) + 1 \text{ for each odd set } S \\
x_e &\geq l_e \text{ for all } e \in E \\
x_e & \quad \text{integral for all } e \in E.
\end{aligned}$$

b-Joins

- Let $G = (V, E)$ be an undirected graph, and let $T \subseteq V$ with $|T|$ even.
- A T*-join* of G is a vector $x \in \mathbb{Z}_+^E$ if for each $v \in V$, $x(\delta_E(v))$ is odd if and only if $v \in T$.
- Let $b \in \mathbb{Z}^V$ be a vector with $b(V)$ even, and let $T = \{v \in V : b_v$ is odd$\}$. Note that $|T|$ is even.
- A b*-join* of G is a vector $x \in \mathbb{Z}_+^E$ if x is a T-join of G, and $x(\delta_E(v)) \geq b_v$ for all $v \in V$.

Minimum b-Joins

- Minimum b-join problem is a relaxation of edges postman problem.
- The corresponding polyhedron has integral extreme points.
- Minimum b-join problem can be solved in polynomial time.

Conclusions and Further Work

- Strenghten the complexity results for the edges postman problem.
- Find better approximation algorithms for the edges postman problem.
- Find a combinatorial algorithm for the minimum b-join problem.