# Approximation Algorithms for a Restricted Mixed Postman Problem

### *CombinaTexas*

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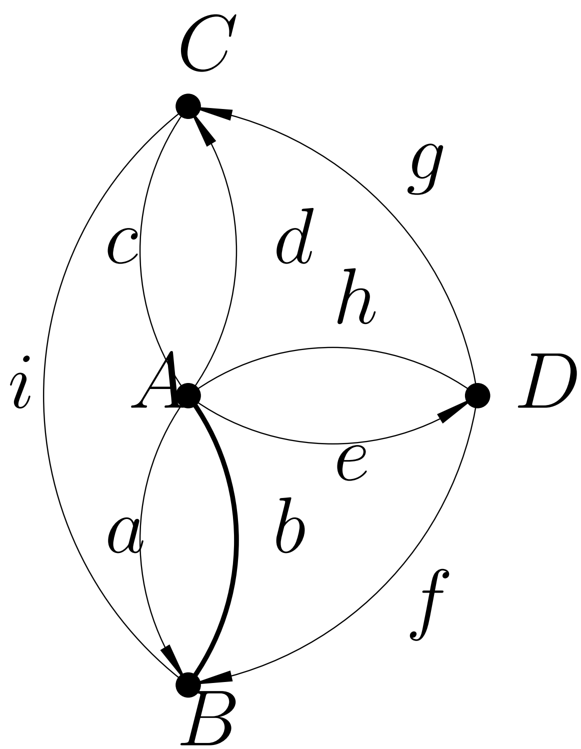
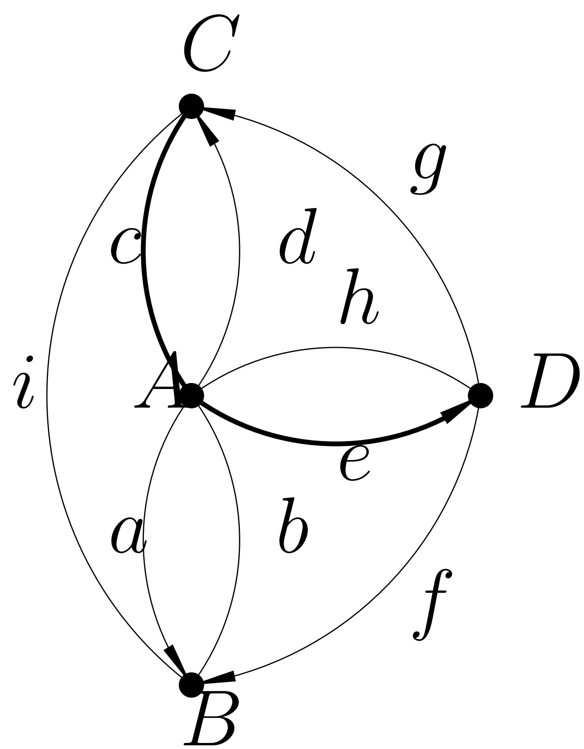
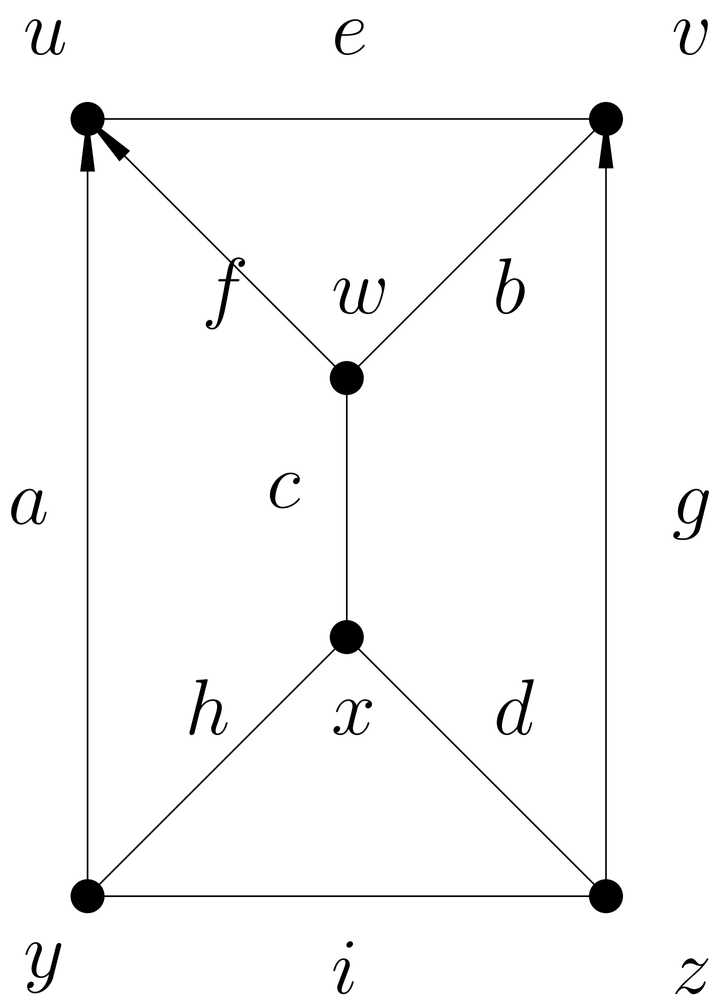
# Mixed graphs and postman tours

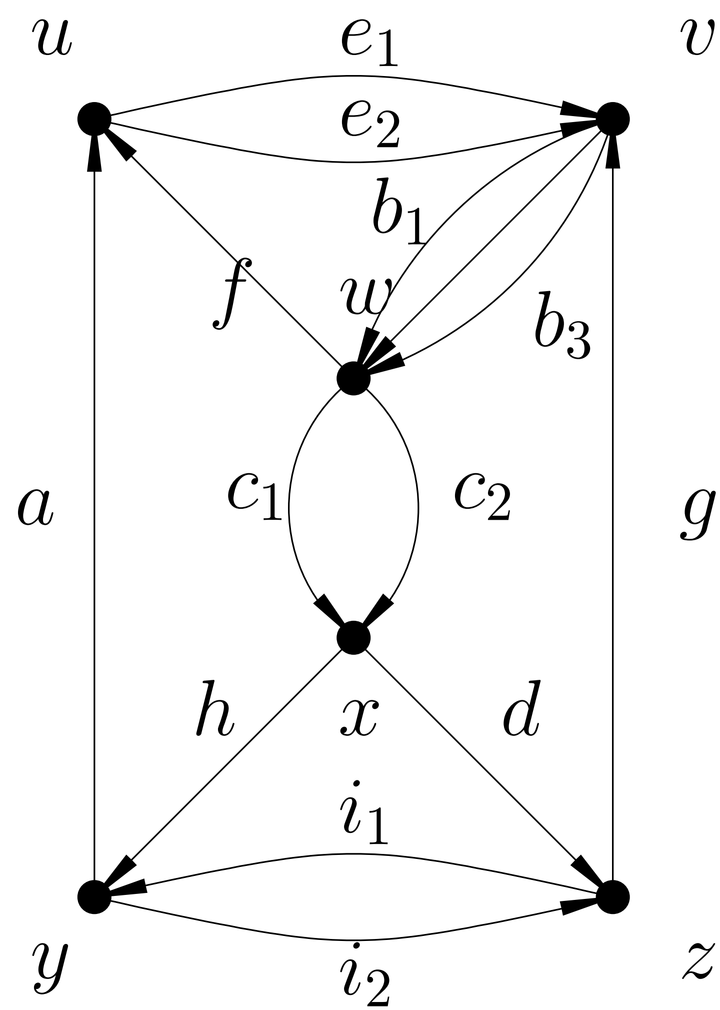
- A mixed graph $M = (V, E, A)$, where $V$ are vertices, $E$ are edges, and $A$ are arcs. A postman tour is closed and visits all of them.



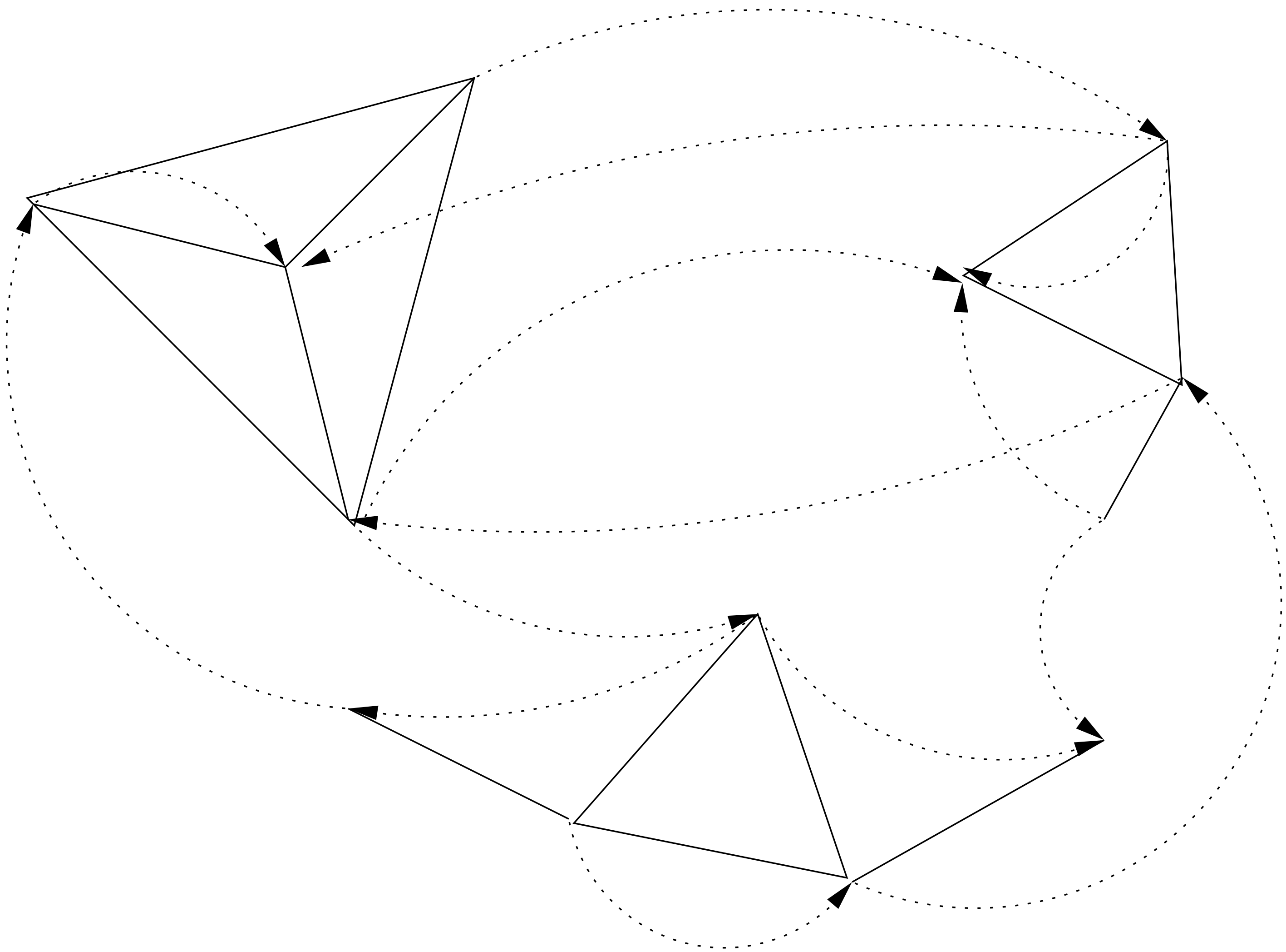



# Postman tours with arc restrictions

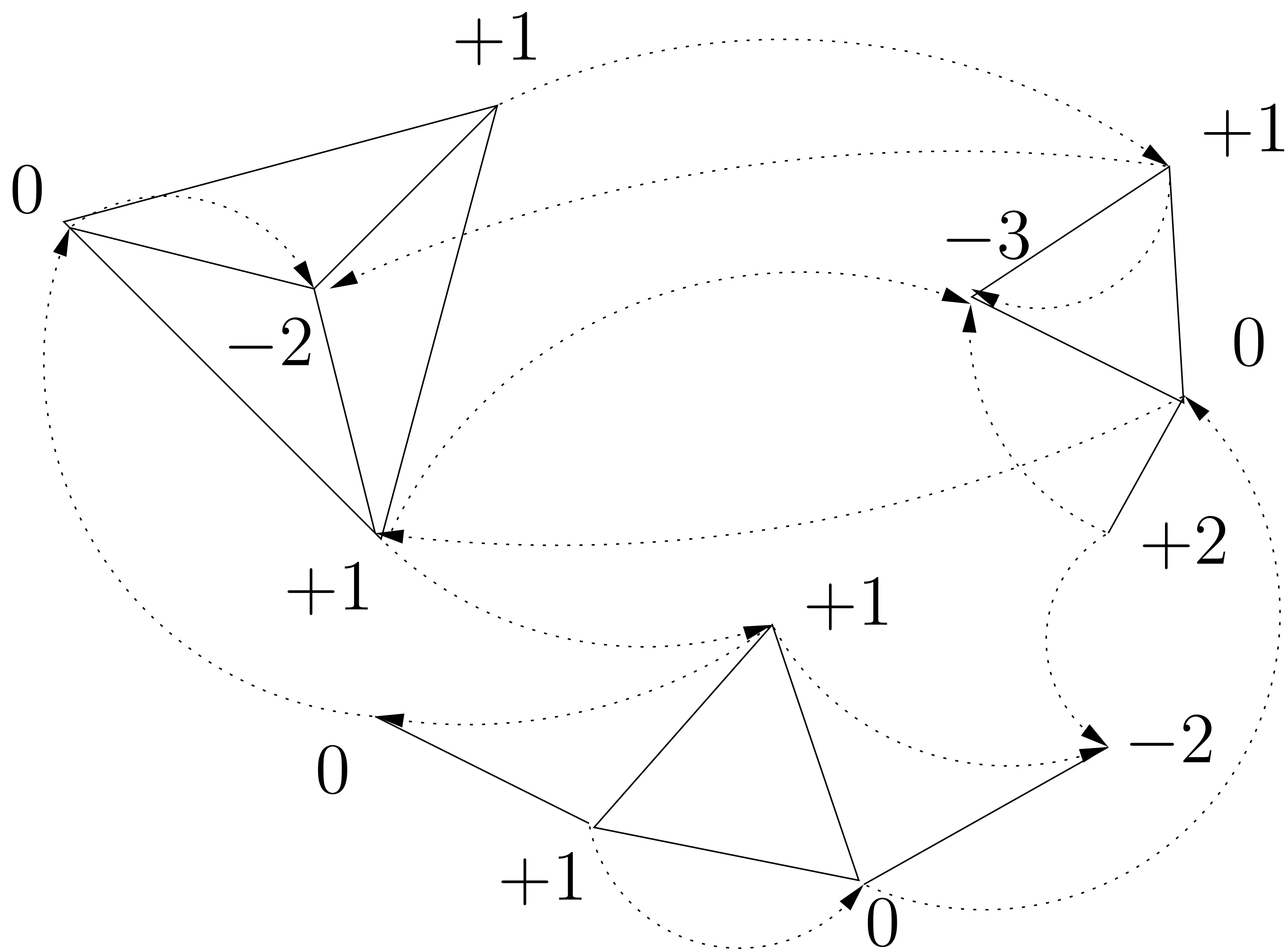


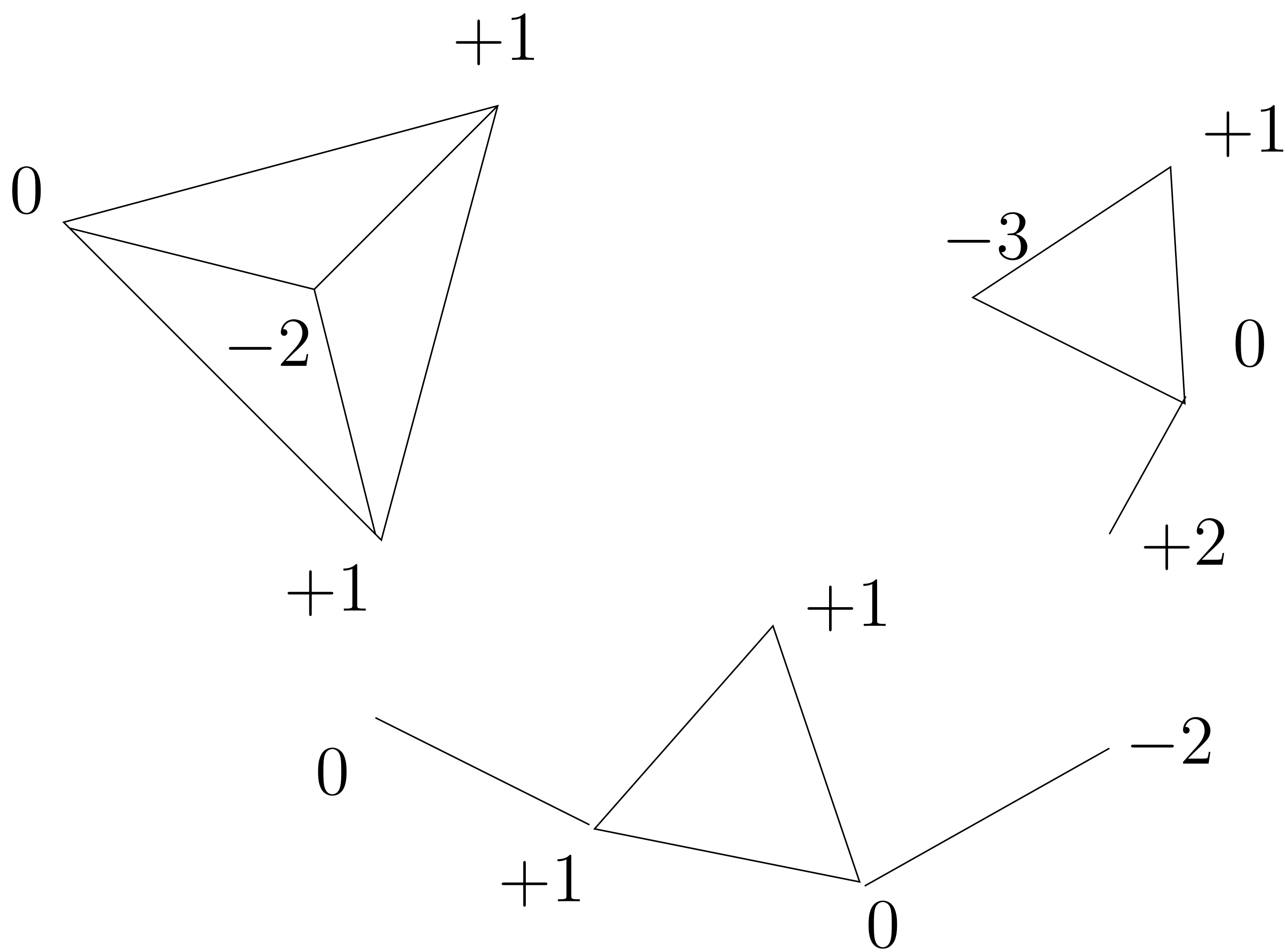
# Transformation (I)



# Transformation (II)

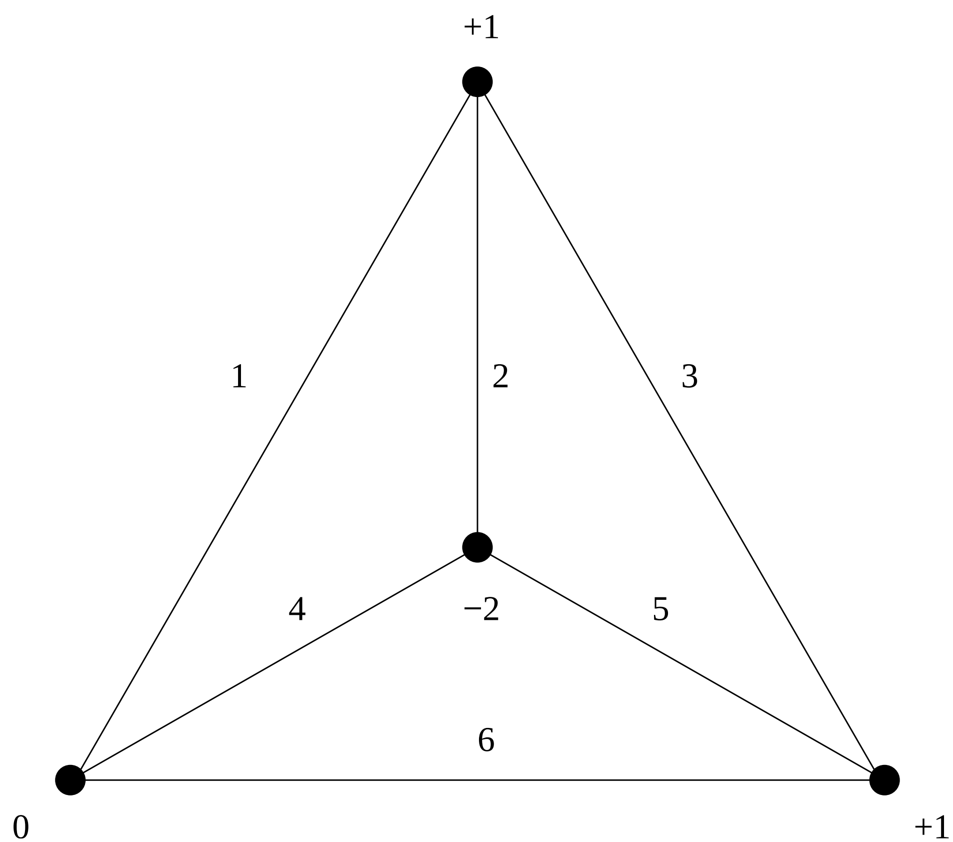


# Transformation (III)



# Flows in undirected graphs

Let $G = (V, E)$ be undirected and $2$-edge connected, $b \in Z^V$ with $b(V) = 0$, and $c \in R_+^E$.



# Bad news

- Finding a minimum cost undirected flow is NP-hard (conjectured by Veerasamy).
- This is still true even if we consider only planar graphs.

# $T$-joins

- Let $G = (V, E)$ and $T \subseteq V$ with $|T|$ even, then $J \subseteq E$ is a $T$-join if $d_J(v)$ is odd iff $v \in T$.
- Minimum cost $T$-joins can be found in polynomial time.
- If $G$ is $2$-edge connected, there exists a $T$-join $J$ such that $c(J) \leq \frac{1}{2}c(E)$.

# Veerasamy's algorithm

- Find a minimum cost directed flow $F$.
- Add unused edges $U$.
- Let $T = \{v \in V : d_U(v)$ is odd$\}$.
- Add a minimum cost $T$-join $J$.
- $C_F \leq C^*$, $C_U \leq C_E \leq C^*$, $C_J \leq \frac{C_E}{2} \leq \frac{C^*}{2}$.
- Therefore $C_1 \leq \frac{5}{2}C^*$.

# Our algorithm

- Let $T = \{v \in V : b_v + d_E(v)$ is odd$\}$.
- Add a minimum cost $T$-join $J$.
- Solve exactly the problem for $G' = (V, E \cup J)$.
- $C_J \leq \frac{1}{2} C_E$.
- $C^* \geq C_E + C_J \geq 3C_J$.
- Therefore $C_4 \leq C^* + C_J \leq \frac{4}{3} C^*$.

# Conclusions

- We show the problem is hard for planar graphs. Are there any polynomially time solvable classes of graphs?
- We improved the guarantee from $\frac{5}{2}$, to $2$, to $\frac{3}{2}$, and to $\frac{4}{3}$. Can we do any better? How about for planar graphs?
- Our algorithm is based on linear programming. Can we give a combinatorial algorithm?