1.14 Express the vector $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$ in the primed triad $\mathbf{i}'\mathbf{j}'\mathbf{k}'$ in which the $x'y'$-axes are rotated about the z-axis (which coincides with the z'-axis) through an angle of 30°.

1.15 Consider two Cartesian coordinate systems xyz and $x'\ y'\ z'$ that initially coincide. The $x'\ y'\ z'$ undergoes three successive counterclockwise 45° rotations about the following axes: first, about the fixed z-axis; second, about its own x'-axis (which has now been rotated); finally, about its own z'-axis (which has also been rotated). Find the components of a unit vector $\mathbf{X}$ in the xyz coordinate system that points along the direction of the x'-axis in the rotated $x'\ y'\ z'$ system. (*Hint: It would be useful to find three transformation matrices that depict each of the above rotations. The resulting transformation matrix is simply their product.*)

1.16 A racing car moves on a circle of constant radius b. If the speed of the car varies with time t according to the equation $v = ct$, where c is a positive constant, show that the angle between the velocity vector and the acceleration vector is 45° at time $t = \sqrt{b/c}$. (*Hint: At this time the tangential and normal components of the acceleration are equal in magnitude.*)

1.17 A small ball is fastened to a long rubber band and twirled around in such a way that the ball moves in an elliptical path given by the equation

$$\mathbf{r}(t) = \mathbf{i}b\cos\omega t + \mathbf{j}2b\sin\omega t$$

where b and ω are constants. Find the speed of the ball as a function of t. In particular, find v at $t = 0$ and at $t = \pi/2\omega$, at which times the ball is, respectively, at its minimum and maximum distances from the origin.

1.18 A buzzing fly moves in a helical path given by the equation

$$\mathbf{r}(t) = \mathbf{i}b\sin\omega t + \mathbf{j}b\cos\omega t + \mathbf{k}ct^2$$

Show that the magnitude of the acceleration of the fly is constant, provided b, ω, and c are constant.

1.19 A bee goes out from its hive in a spiral path given in plane polar coordinates by

$$r = be^{kt} \qquad \theta = ct$$

where b, k, and c are positive constants. Show that the angle between the velocity vector and the acceleration vector remains constant as the bee moves outward. (*Hint: Find* $\mathbf{v}\cdot\mathbf{a}/va$.)

1.20 Work Problem 1.18 using cylindrical coordinates where $R = b$, $\phi = \omega t$, and $z = ct^2$.

1.21 The position of a particle as a function of time is given by

$$\mathbf{r}(t) = \mathbf{i}(1 - e^{-kt}) + \mathbf{j}e^{kt}$$

where k is a positive constant. Find the velocity and acceleration of the particle. Sketch its trajectory.

1.22 An ant crawls on the surface of a ball of radius b in such a manner that the ant's motion is given in spherical coordinates by the equations

$$r = b \qquad \phi = \omega t \qquad \theta = \frac{\pi}{2}\left[1 + \frac{1}{4}\cos(4\omega t)\right]$$

Find the speed of the ant as a function of the time t. What sort of path is represented by the above equations?

1.23 Prove that $\mathbf{v} \cdot \mathbf{a} = v\dot{v}$ and, hence, that for a moving particle **v** and **a** are perpendicular to each other if the speed v is constant. (*Hint: Differentiate both sides of the equation* $\mathbf{v} \cdot \mathbf{v} = v^2$ *with respect to* t. *Note,* $\dot{v}$ *is not the same as* $|\mathbf{a}|$. *It is the magnitude of the acceleration of the particle along its instantaneous direction of motion.*)

1.24 Prove that

$$\frac{d}{dt}[\mathbf{r} \cdot (\mathbf{v} \times \mathbf{a})] = \mathbf{r} \cdot (\mathbf{v} \times \dot{\mathbf{a}})$$

1.25 Show that the tangential component of the acceleration of a moving particle is given by the expression

$$a_\tau = \frac{\mathbf{v} \cdot \mathbf{a}}{v}$$

and the normal component is therefore

$$a_n = \left(a^2 - a_\tau^2\right)^{1/2} = \left[a^2 - \frac{(\mathbf{v} \cdot \mathbf{a})^2}{v^2}\right]^{1/2}$$

1.26 Use the above result to find the tangential and normal components of the acceleration as functions of time in Problems 1.18 and 1.19.

1.27 Prove that $|\mathbf{v} \times \mathbf{a}| = v^3/\rho$, where ρ is the radius of curvature of the path of a moving particle.

1.28 A wheel of radius b rolls along the ground with constant forward acceleration a_0. Show that, at any given instant, the magnitude of the acceleration of any point on the wheel is $(a_0^2 + v^4/b^2)^{1/2}$ relative to the center of the wheel and is also $a_0[2 + 2\cos\theta + v^4/a_0^2b^2 - (2v^2/a_0b)\sin\theta]^{1/2}$ relative to the ground. Here v is the instantaneous forward speed, and θ defines the location of the point on the wheel, measured forward from the highest point. Which point has the greatest acceleration relative to the ground?

1.29 What is the value of x that makes of following transformation **R** orthogonal?

$$\mathbf{R} = \begin{pmatrix} x & x & 0 \\ -x & x & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

What transformation is represented by **R**?

1.30 Use vector algebra to derive the following trigonometric identities

(**a**) $\cos(\theta - \phi) = \cos\theta\cos\phi + \sin\theta\sin\phi$

(**b**) $\sin(\theta - \phi) = \sin\theta\cos\phi - \cos\theta\sin\phi$