

- 1.2. Show for a solid spherical ball of mass  $m$  rotating about an axis through its center with a charge  $q$  uniformly distributed on the surface of the ball that the magnetic moment  $\mu$  is related to the angular momentum  $\mathbf{L}$  by the relation

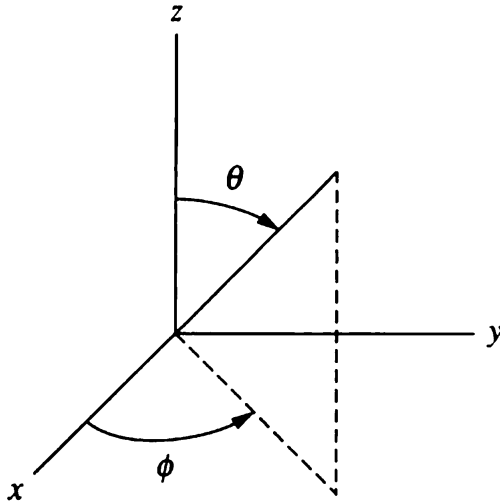
$$\mu = \frac{5q}{6mc} \mathbf{L}$$

*Reminder:* The factor of  $c$  is a consequence of our using Gaussian units. If you work in SI units, just add the  $c$  in by hand to compare with this result.

- 1.3. In Problem 3.2 we will see that the state of a spin- $\frac{1}{2}$  particle that is spin up along the axis whose direction is specified by the unit vector  $\mathbf{n} = \sin \theta \cos \phi \mathbf{i} + \sin \theta \sin \phi \mathbf{j} + \cos \theta \mathbf{k}$ , with  $\theta$  and  $\phi$  shown in Fig. 1.11, is given by

$$|+\mathbf{n}\rangle = \cos \frac{\theta}{2} |+\mathbf{z}\rangle + e^{i\phi} \sin \frac{\theta}{2} |-\mathbf{z}\rangle$$

- (a) Verify that the state  $|+\mathbf{n}\rangle$  reduces to the states  $|+\mathbf{x}\rangle$  and  $|+\mathbf{y}\rangle$  given in this chapter for the appropriate choice of the angles  $\theta$  and  $\phi$ .  
 (b) Suppose that a measurement of  $S_z$  is carried out on a particle in the state  $|+\mathbf{n}\rangle$ . What is the probability that the measurement yields (i)  $\hbar/2$ ? (ii)  $-\hbar/2$ ?  
 (c) Determine the uncertainty  $\Delta S_z$  of your measurements.



**FIGURE 1.11**

The angles  $\theta$  and  $\phi$  specifying the orientation of an  $S_{\mathbf{n}}$  device.

- 1.4. Repeat the calculations of Problem 1.3 (b) and (c) for measurements of  $S_x$ . *Hint:* Infer what the probability of obtaining  $-\hbar/2$  for  $S_x$  is from the probability of obtaining  $\hbar/2$ .  
 1.5. (a) What is the amplitude to find a particle that is in the state  $|+\mathbf{n}\rangle$  (from Problem 1.3) with  $S_y = \hbar/2$ ? What is the probability? Check your result by evaluating the probability for an appropriate choice of the angles  $\theta$  and  $\phi$ .  
 (b) What is the amplitude to find a particle that is in the state  $|+\mathbf{y}\rangle$  with  $S_n = \hbar/2$ ? What is the probability?  
 1.6. Show that the state

$$|-\mathbf{n}\rangle = \sin \frac{\theta}{2} |+\mathbf{z}\rangle - e^{i\phi} \cos \frac{\theta}{2} |-\mathbf{z}\rangle$$

satisfies  $\langle +\mathbf{n} | -\mathbf{n} \rangle = 0$ , where the state  $|+\mathbf{n}\rangle$  is given in Problem 1.3. Verify that  $\langle -\mathbf{n} | -\mathbf{n} \rangle = 1$ .

1.7. A beam of spin- $\frac{1}{2}$  particles is sent through a series of three Stern-Gerlach measuring devices, as illustrated in Fig. 1.12. The first SGz device transmits particles with  $S_z = \hbar/2$  and filters out particles with  $S_z = -\hbar/2$ . The second device, an SGn device, transmits particles with  $S_n = \hbar/2$  and filters out particles with  $S_n = -\hbar/2$  where the axis  $\mathbf{n}$  makes an angle  $\theta$  in the  $x$ - $z$  plane with respect to the  $z$  axis. Thus particles after passage through this SGn device are in the state  $|+\mathbf{n}\rangle$  given in Problem 1.3 with the angle  $\phi = 0$ . A last SGz device transmits particles with  $S_z = -\hbar/2$  and filters out particles with  $S_z = \hbar/2$ .

- What fraction of the particles transmitted by the first SGz device will survive the third measurement?
- How must the angle  $\theta$  of the SGn device be oriented so as to maximize the number of particles that are transmitted by the final SGz device? What fraction of the particles survive the third measurement for this value of  $\theta$ ?
- What fraction of the particles survive the last measurement if the SGn device is simply removed from the experiment?

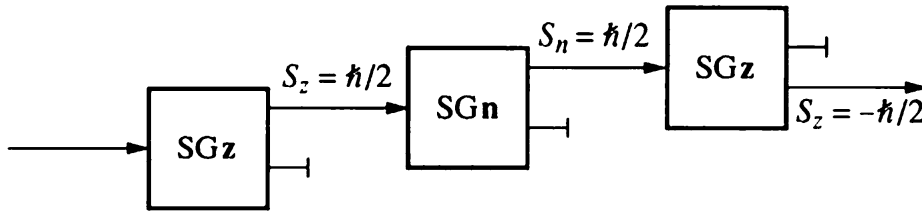


FIGURE 1.12

1.8. The state of a spin- $\frac{1}{2}$  particle is given by

$$|\psi\rangle = \frac{i}{\sqrt{3}}|+\mathbf{z}\rangle + \sqrt{\frac{2}{3}}|-\mathbf{z}\rangle$$

What are  $\langle S_z \rangle$  and  $\Delta S_z$  for this state? Suppose that an experiment is carried out on 100 particles, each of which is in this state. Make up a reasonable set of data for  $S_z$  that could result from such an experiment. What if the measurements were carried out on 1,000 particles? What about 10,000?

1.9. Show that neither the probability of obtaining the result  $a_i$  nor the expectation value  $\langle A \rangle$  are affected by  $|\psi\rangle \rightarrow e^{i\delta}|\psi\rangle$ , that is, by an overall phase change for the state  $|\psi\rangle$ .