Problema de Control Óptimo sobre un sistema de PDE

$$\text{PCL} \quad \begin{cases} v^0 \in U = L^2((0,T),\ \mathsf{R})^M \\ \mathsf{J}(v) \leq\ \mathsf{J}(u), \forall u \in U \end{cases}$$

donde

$$\mathsf{J}(v) = \frac{1}{2}\int_0^T ||v||^2 \mathrm{d}t + \frac{k_1}{2}\int_0^L ||y(T) - z_0||^2 \mathrm{d}x + \frac{k_2}{2}\int_0^L ||\varphi_x||^2 \mathrm{d}x,$$

$$y_{tt} - y_{xx} + y^3 = f + \sum_{j=1}^{M} v_j \delta(x - a_j), \quad \text{(EDy)}$$

$$L > 0, z_0 : [0,L] \to \mathsf{R},$$

$$y(0,t) = y(L,t) = 0,$$

$$y(0) = y_0, \qquad z_1 : [0,L] \to \mathsf{R}, y_1 : [0,L] \to \mathsf{R},$$

$$y_t(x,0) = y_1(x)$$

$$a_j \in [0,L], j = 1, \ldots, M$$

$$-\varphi_{xx} = y_t(T) - z_1, \quad \text{(EDz)}$$

$$\varphi(0) = \varphi(L) = 0$$